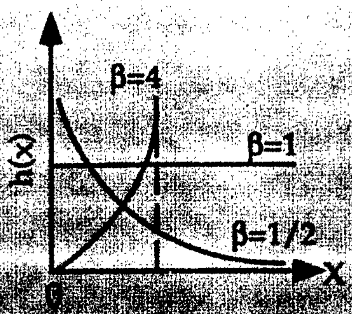

Mechanical Applications in Reliability Engineering

Reliability Analysis Center

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Reliability engineering practices are presented from two primary perspectives - part reliability engineering and system reliability engineering. Unlike many reliability documents, this document establishes a clear distinction between the tools and terminology which are associated with each perspective. The context is oriented toward mechanical reliability applications within reliability engineering. Such topics as: the mechanisms of mechanical failure, mechanical failure theories, part failure data evaluation, system failure data evaluation and failure mode evaluation tools for systems are discussed.

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Mechanical Reliability	NHPP
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PREFACE

Objective: This is a document intended for those who require a specific knowledge of reliability theory and principles as applied to mechanical parts and systems. The content of this document was selected based on a single criteria - that of utility. The most practical reliability tools which are currently being effectively applied to assure reliable systems are discussed. RAC authors have selected and organized the topics contained in this document based on the anticipated needs of those who must understand and apply the concepts of reliability to system design and development. It is our goal to provide the reliability practitioner with a concise document which contains only the most useful and theoretically accurate tools to assure reliable systems. We hope we have accomplished this goal in a manner which will provide long lasting benefit to those who take the time to read and understand this material.

Background: This document is the first revision (second edition) to the RAC document, "Analysis Techniques For Mechanical Reliability" which was first published in 1985. Many enhancements have been added, especially in the area of repairable system reliability. Many of the statistical analysis tools presented in the original document still remain in this first revision. Many have been updated and expanded to include further examples. These examples aid in self-study and promote an understanding of the utility of reliability evaluation techniques. The reader should feel free to supplement the examples provided in the document with his or her own unique experiences. In this way, the reader can customize this document to specific systems of personnel interest.

Content: The content of this document has been separated into four major sections:

Section A: Introduction to Reliability
Section B: Fundamental Statistical Concepts
Section C: Part Reliability Engineering
Section D: System Reliability Engineering

Within each major section, numerous concepts or topics have been discussed as detailed in the table of contents for this document. The material contained in each section is intended to build upon the material of the previous section(s). In order to develop well rounded reliability analysis skills, it is recommended that this document be read from cover to cover at least once and then used as a reference source for future tasks. Knowledge or insight gained from Section A (Introduction) regarding part and system concepts/terminology will be applied throughout Section C (Part Reliability Engineering) and Section D (System Reliability Engineering) where detailed reliability modeling and evaluation techniques are discussed. Much of the terminology which is presented in Section A is the same terminology which is accepted and utilized by others in the reliability community and will hopefully be familiar to you. Essential statistical concepts are presented in Section B. Section B provides insight into the random variables of reliability and how they are described. This will be vital to understanding other topics presented throughout this document.

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- Patrick Hetherington and Richard Uncle for technical input
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- Pam Meus and Jeanne Crowell for clerical support
- Perry Nichols for supplying necessary engineering documents

Their support is greatly appreciated. Thank you.

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INTRODUCTION TO RELIABILITY

1.0 INTRODUCTION

Reliability engineering is a professional discipline which combines knowledge in statistics and engineering for the purpose of quantitatively evaluating, predicting, measuring and improving the reliability of products. Reliability engineering procedures have been applied to a vast array of products, some of which include: machines (of all types), structures, computer software and materials to name just a few. What makes reliability engineering or any other engineering discipline usable for its practitioners are the analysis tools which are generated from the collective knowledge assembled within the discipline.

The material contained in this document emphasizes the various reliability analysis techniques which are available to those who must evaluate, model or predict the reliability of parts and systems. Although mechanical applications are emphasized in this document, many of the theories which are presented can be universally applied to other functional areas. It must be realized that these reliability analysis techniques only provide the means to an end. These analysis tools will prove useful as justification for the design changes, corrective actions and planning decisions which directly improve product reliability. As reliability practitioners, we should strive to provide the best justification possible when recommending design changes or planning future activities based on the expected performance of a product.

With this in mind, this document was developed by RAC engineers to meet the specific goals identified in Figure 1.0-1 and to provide a well rounded discussion of both part and system reliability analysis tools. The major sections of this document are the following:

Section A: Introduction To Reliability

Section B: Fundamental Statistical Concepts

Section C: Part Reliability Engineering

Section D: System Reliability Engineering

SECTION A: INTRODUCTION TO RELIABILITY

Goals:
- Establish part and system reliability engineering perspectives
- Define a "part"
- Define a "system"
- Establish basic terminology associated with each perspective

↓

SECTION B: FUNDAMENTAL STATISTICAL CONCEPTS

Goals:
- Establish statistical concepts & terminology pertinent to reliability
- Discuss concepts of: the random variable, sample, population, probability, distribution of a random variable, and dispersion and central tendency numerics associated with a random variable
- Identify the random variables of reliability

↓

SECTION C: PART RELIABILITY ENGINEERING

Goals:
- Discuss mechanical failure mechanisms
- Discuss mechanical failure theories and their utility
- Establish distribution statistics as a primary part failure data analysis procedure with emphasis on the Weibull failure model
- Identify and discuss the utility of other part reliability evaluation techniques such as interference analysis and life assessment models

↓

SECTION D: SYSTEM RELIABILITY ENGINEERING

Goals:
- Establish the point process approach to system failure modeling
- Discuss important point process models such as Homogeneous Poisson Process (HPP) and Nonhomogeneous Poisson Process (NHPP)
- Discuss utility of failure mode evaluation tools such as Failure Mode, Effects and Criticality Analysis (FMECA) and Fault Tree Analysis (FTA)

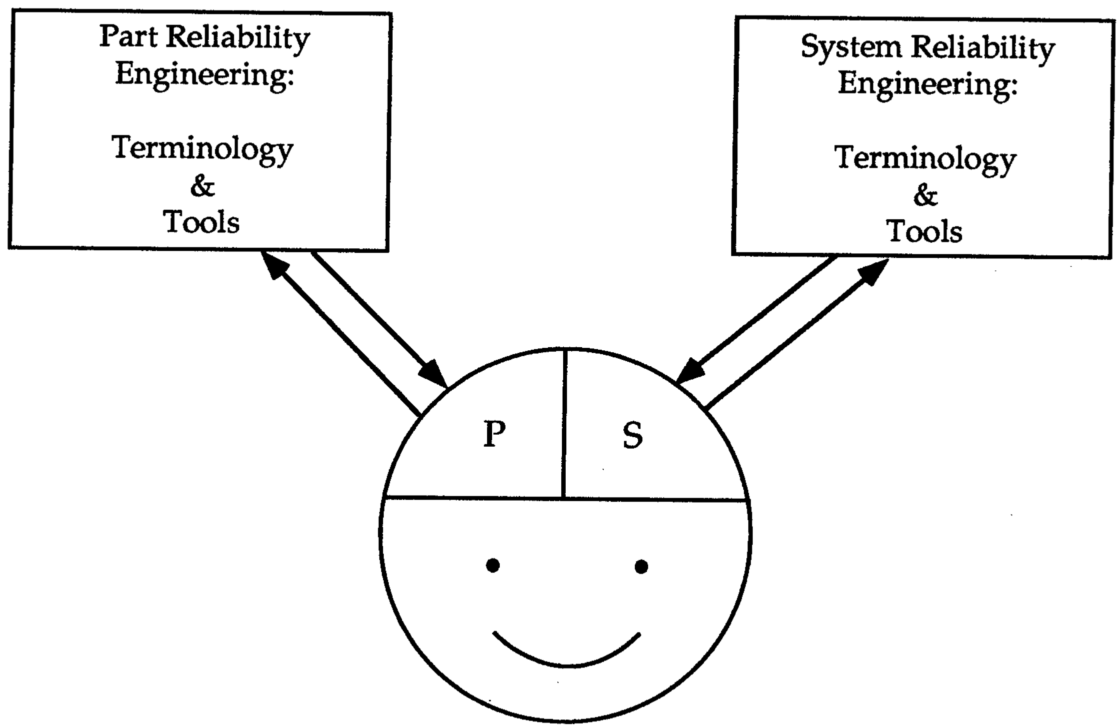
FIGURE 1.0-1: SUMMARY OF DOCUMENT GOALS

1.1 Reliability Engineering Perspectives

Reliability engineering has developed two principal perspectives toward the analysis of reliability. These two principal perspectives are: part and system reliability engineering. Each perspective has evolved in order to evaluate different empirical and analytical reliability issues. Each also deals with different items of primary interest. Part reliability is concerned with the failure characteristics of the individual nonrepairable part to make inferences about the part population. System reliability is concerned with the failure characteristics of a group of typically different parts assembled as a repairable system. Past history has shown that the analysis of parts and the use of part reliability based theories has dominated the reliability discipline. Unfortunately, in some cases, this practice has led to the misapplication of part reliability theories to systems. But, this domination still exists even though many of the system reliability theories were well documented as far back as the mid-1960s (Reference [59]).

It is essential to realize that *two* principal perspectives exist and represent paths of analytical diversity within the study of reliability. Each perspective offers its own unique set of reliability terminology and statistical theories. It is the goal of this document to provide the reader with an accurate account of each perspective and to focus on the use of appropriate terminology and analysis techniques when evaluating parts and/or systems.

Reliability analysis techniques and terminology are not universally applicable but are a function of which perspective is in effect; part or system. In general, a segregated thought process, as shown in Figure 1.1-1, will serve the novice best when developing, interpreting, or communicating reliability information. There are numerous examples in reliability literature where incorrect evaluations have been performed because of confusion and misuse of reliability analysis techniques and terminology. Many of these misapplications could have been prevented with a better understanding of the differences between part and system evaluation procedures. Examples of these misapplications are identified and discussed in Reference [55].



"The Novice"

FIGURE 1.1-1: A "NOVICE" APPROACH TO RELIABILITY ENGINEERING CONCEPTS

Approaching a reliability task from the correct perspective is required to obtain valid results. It will also improve your ability to communicate the results to colleagues or the general reliability community. Figure 1.1-2 illustrates this concept and shows efficient lines of communication among the various members of the reliability community. Each member is enjoying a balanced perspective toward both part and system reliability.

Once the correct perspective has been established, the correct terminology can be applied. For example, one collects individual time-to-part failure (TTF) data for parts but collects time-between-successive system failures (TBF) data for a system. Understanding these subtle differences in terminology will improve our ability to develop, interpret and communicate reliability information. Figure 1.1-3 illustrates some common terms which are associated with either part or system reliability. All of the topics indicated in Figure 1.1-3 are discussed at various points throughout this document.

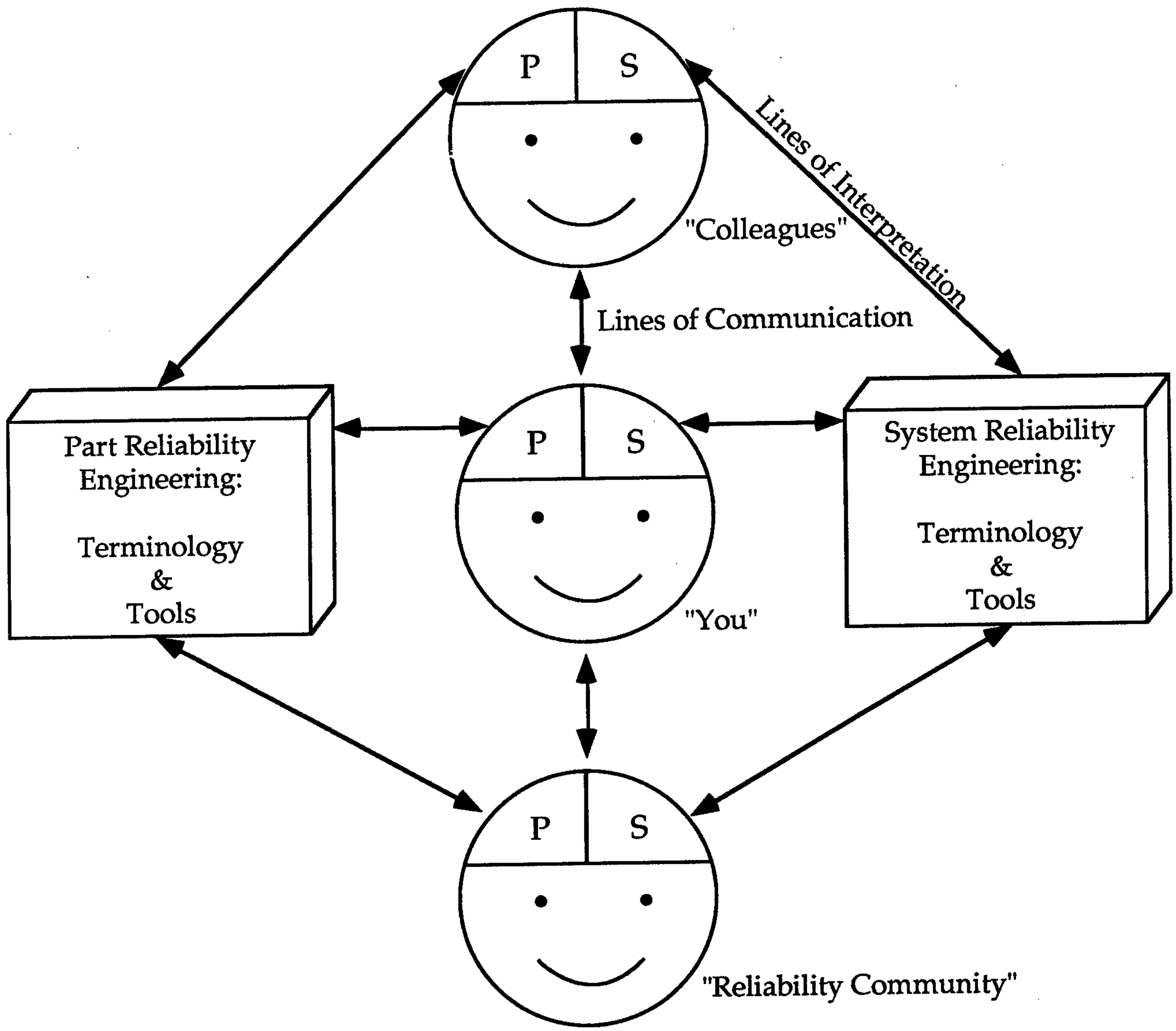
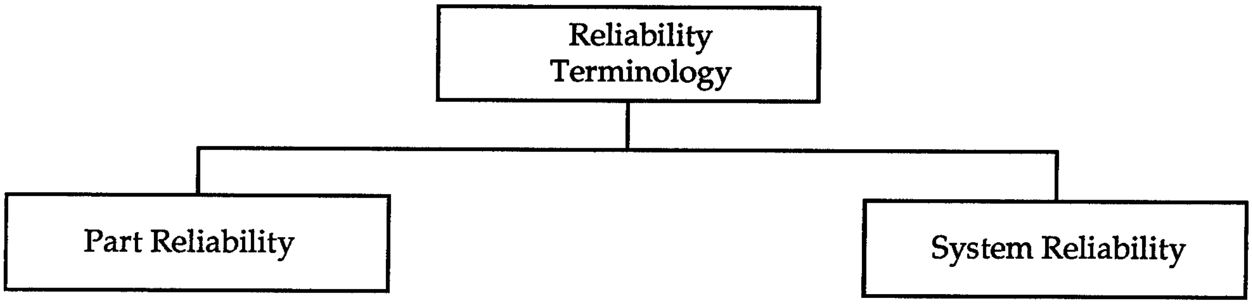


FIGURE 1.1-2: INTERPRETING AND COMMUNICATING RELIABILITY ENGINEERING INFORMATION WITH SUCCESS - HAVE A BALANCED PERSPECTIVE



Part Reliability

- Time-to-Part Failure of the ith part (TTF_i)
- Mean Time to Failure (MTTF)
- Failure Distributions
- Exponential Failure Model
- Weibull Failure Model
- Force of Mortality (FOM) or Hazard Rate, h(x)
- Wearout
- Order Statistics $\left(TTF_{(1)}, TTF_{(2)} \ldots TTF_{(n)}\right)$
- Spacing Between Order Statistics (i.e., the time between ordered TTF)
- "x" for the age of a part

System Reliability

- Time-Between - Successive System Failures $\left(TBF_i\right)$ or Interarrival Times
- Mean Time Between Failure (MTBF)
- Arrival Times $\left(TTSF_i\right)$ or time to the i^{th} system failure
- Failure Process
- Point Process Model
- Homogeneous Poisson Process (HPP)
- Non Homogeneous Poisson Process (NHPP)
- Rate of Occurrence of Failure (ROCOF)
- Deterioration
- Chronological Ordering
- Laplace Trend Test
- "t" for the age of a system

FIGURE 1.1-3: SIGNIFICANT RELIABILITY TERMS AND THEIR ASSOCIATION

1.2 Functional Categories of Reliability

Reliability has been segregated into functional categories. Functional categories have been identified in numerous discussions of reliability and include: mechanical, electronic, structural and materials to name just a few. These categories are significant because they identify specific specialty areas within reliability. They represent a natural transition to more detailed areas of expertise and are typically associated with the standard engineering disciplines such as mechanical and electrical engineering. As illustrated in Figure 1.2-1, these functional areas can be viewed as specialty "spin-off" areas from the main body of reliability engineering.

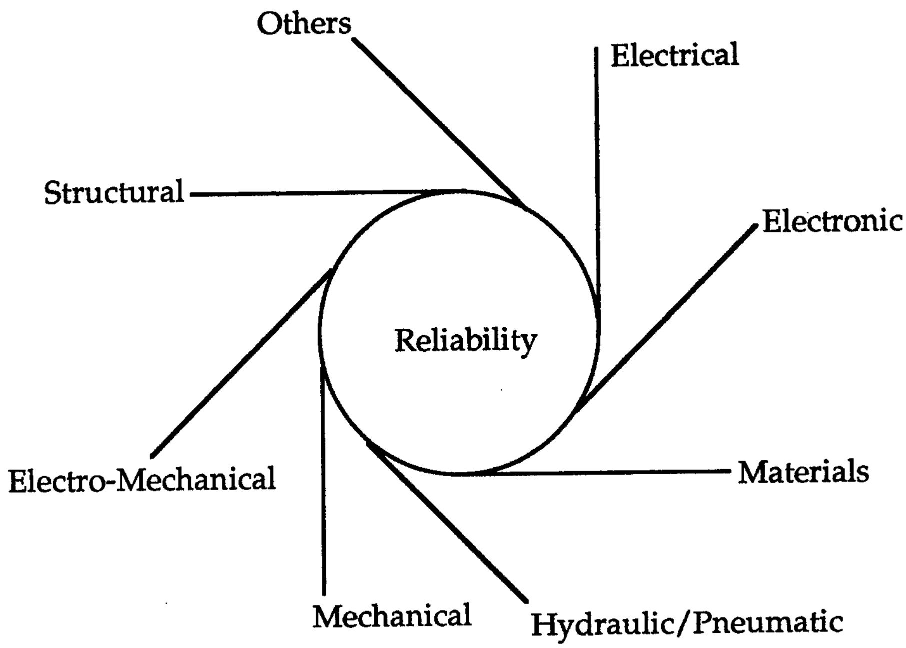


FIGURE 1.2-1: FUNCTIONAL AREAS OF RELIABILITY

The functional areas of reliability have been greatly enhanced because of the enormous knowledge base which currently exists in each of the related engineering disciplines. In order for reliability models to be meaningful, engineering variables must be transitioned into reliability variables such that no engineering theories are violated. In this way, reliability engineering practices will enhance present engineering procedures.

Mechanical reliability is a specific functional area of reliability engineering which specializes in the application of reliability principles to mechanical parts and systems with mechanical parts. Here the analyst can use his or her engineering expertise of mechanical failure mechanisms, mechanical failure theories, material properties, stress concentrations, fatigue theory, fracture mechanics or other related topics to improve the reliability of parts and systems. The functional area of mechanical reliability will be emphasized throughout this document in the form of mechanically oriented discussions and examples.

1.3 Concept of a Part

Essential to the understanding of material contained in this document is the significance of what characterizes a "part" and what characterizes a "system". A "part" will be defined as a nonrepairable item which can only fail once and is then discarded. A part can be categorized as a simple part or a complex part. A simple part consists of a single component. For example, o-rings, belts, bolts, springs or gears can be considered simple parts. A complex part consists of more than one component. For example, a ball bearing, relay, thermostat, fuse or spark plug can be considered complex parts because, upon failure they are typically discarded, but unlike simple parts, do contain multiple components.

Figure 1.3-1 graphically portrays the time-to-failure (TTF) associated with a group of identical parts tested to failure. Note how parts exhibit no life after first failure and are considered nonrepairable.

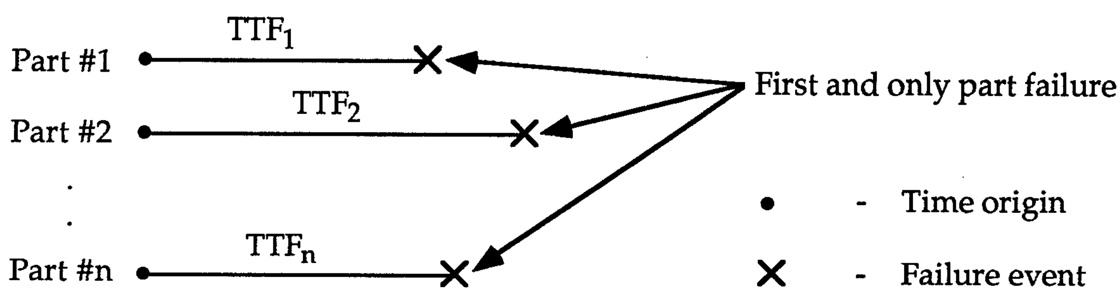


FIGURE 1.3-1: ILLUSTRATION OF PART FAILURE FOR A SAMPLE OF N IDENTICAL PARTS

Probabilistic failure models ($f_P(x)$, $F_P(x)$, $R_P(x)$) can be generated from a sample of identical parts which have failed such as those illustrated in Figure 1.3-1. These probabilistic models define the reliability characteristics of each part in the sample. As the sample size, n, approaches infinity (∞), the reliability characteristics approach their true values. The true values being representative of the entire part population. The probabilistic models associated with TTF data are discussed in Section B and the procedures for probabilistic modeling of TTF data are discussed in detail in Section 6.0.

1.4 Concept of a System

Unlike a part, a system has a particular characteristic which makes it unique, namely, the ability to experience successive failure events over its lifetime. This sequential series of failure events can be illustrated on a continuous time line as shown in Figure 1.4-1 and is called a failure process.

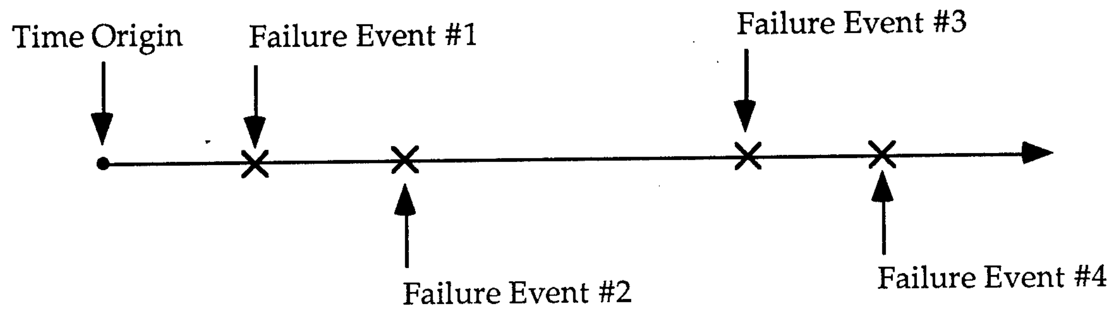
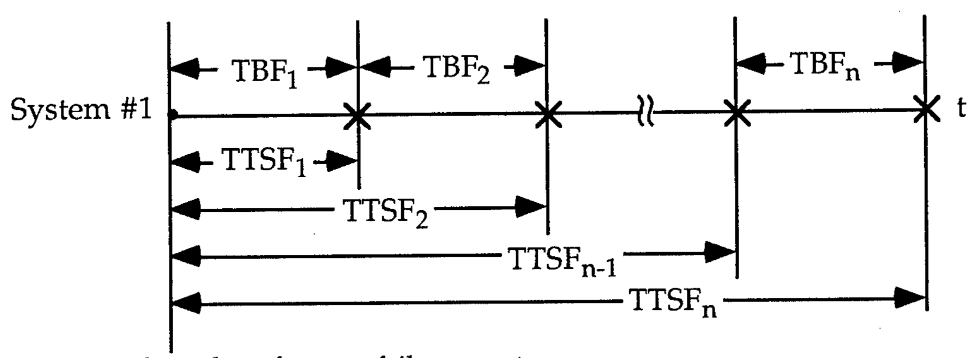


FIGURE 1.4-1: SYSTEM FAILURE PROCESS: TIME LINE OF SYSTEM FAILURE EVENTS

The time line is typically representative of system operating time. Comparing Figure 1.3-1 and Figure 1.4-1 reveals the differences between part failures and system failures, namely, a part can only fail once but a system can fail numerous times. This significant difference will form the basis for further statistical pursuit within each of these areas.

The notation given to the various time segments of the system failure process is identified in Figure 1.4-2.



n = total number of system failure events
TBF_i = time between successive system failure events or interarrival times
$TTSF_i$ = time to system failure events or arrival times

FIGURE 1.4-2: TIME SEGMENT NOTATION FOR SYSTEM FAILURE PROCESS

Other references on repairable system reliability and time series analysis have used other notation for both arrival times (e.g., T_i and x_i) and interarrival times (ex., X_i). This document has selected a slightly more representative set of notation, as illustrated in Figure 1.4-2, to improve the distinction between part and system variables. This essential notation will also be summarized in the next section.

1.5 Essential Notation

The following is a list which summarizes the most significant notation to be used throughout this document. Some of the notation has already been introduced in previous sections and should be reviewed again at this time. Much of this terminology is consistent with other published works and this will hopefully improve the utility of the material to follow. A glossary of reliability terms is also provided in Appendix A.

Parts

P	-	represents the random variable: time to part failure, a primary random variable in part reliability engineering
$\{TTF_1, TTF_2 \ldots TTF_n\}$	-	set of time to part failure data from n identical parts
$TTF_{(1)}, TTF_{(2)} \ldots TTF_{(n)}$	-	time to failure data from n identical parts which is ordered by magnitude such that $TTF_{(1)} \leq TTF_{(2)} \ldots \leq TTF_{(n)}$ (i.e., order statistics)
$TTF_{(i+1)} - TTF_{(i)}$	-	spacing between ordered time to part failure data
x	-	age of a part, typically considered the operational age
$f_P(x)$	-	density function which describes a set of time to part failure data from n identical parts where $n \rightarrow \infty$
$F_P(x)$	-	cumulative distribution function which describes a set of time to part failure data from n identical parts where $n \rightarrow \infty$
$h_P(x)$	-	hazard rate or force of mortality (FOM) of time to part failure
$R_P(x)$	-	part reliability or probability that the part survives to age x

Systems

S_i	-	represents a system random variable which is the time between the (i-1)st and ith system failures extracted from a population of system failure processes
TBF_i	-	time between the (i-1)st and ith system failures, also the ith interarrival time of one system failure process
T_i	-	represents a system random variable which is the time to the ith system failure or the ith arrival time
$TTSF_i$	-	time to the ith system failure also the ith arrival time from one system failure process. A value of random variable T_i
t	-	total system age, typically considered the operational age
TBF_1, TBF_2 ... TBF_n	-	represents the natural order of interarrival times of one system failure process which has "n" failure events

General

f(x)	-	probability density function of a random variable, X
F(x)	-	cumulative distribution function of a random variable, X
h(x)	-	hazard rate or force of mortality of a random variable, X
R(x)	-	survivor function or reliability function of a random variable, X

SECTION B

FUNDAMENTAL STATISTICAL CONCEPTS

2.0 INTRODUCTION TO STATISTICAL CONCEPTS

There are many statistical tools which are used within reliability, the most basic statistical tools will be discussed in this section. These basic statistical tools are provided here in order to emphasize their importance and to discuss their utility with respect to evaluating parts or systems. Even those people with superficial involvèment in reliability issues should be knowledgeable with respect to the statistical concepts provided in this section. This section will concentrate on the following primary topics:

1. Random variables within reliability
2. Defining part and system reliability
3. Probability functions
4. Continuous statistical distributions
5. Hazard rate

2.1 Concept of a Random Variable

The concept of a random variable is basic to an understanding of statistical theories. A random variable is very simply any variable, such as the life of a part, maximum stress in a part or the time to the first critical failure in a system, whose value cannot be exactly specified for each element in the population. Random variables must be represented probabilistically as opposed to deterministically. The concept of a random variable can be illustrated with the following data set which represents the random variable: cycles-to-failure. The following cycles-to-failure were obtained for a group of 50 identical relays placed on a life test:

1283	4865	8559	14840	30822
1887	5147	8843	14988	31473
1888	5350	9305	16306	35811
2357	5353	9460	17621	38319
3137	5536	9595	17807	41554
3606	6499	10247	20747	42870
3752	6820	11492	21990	50246
3914	7733	12913	23449	62690
4394	8025	12937	28946	63910
4398	8185	13210	29254	73473

By observation of this sample of 50 relays, the random variable exhibits a range from 1283 to 73473 cycles-to-failure. When another relay (from this population of relays) is placed on life test, one would expect the resulting life to fall within this range if the initial 50 samples were representative of the population of relays. What we cannot do is state the exact value of relay life with certainty.

The majority of random variables used in reliability are termed continuous random variables. These are random variables where the number of possible outcomes is infinite. For example, suppose we define a random variable X, as the length of time a certain valve type will operate properly before failure. In this example, the set of possible outcomes for X may be considered to be equal to the number of points on the positive real line ($0 \leq x < \infty$). We could also say that the set of possible outcomes is the whole real line ($-\infty < x < \infty$), even though a negative value can never actually occur when the random variable considered is life. Whenever the range of possibilities is infinite, the set of possible outcomes is said to be continuous, and a random variable defined over this set is called a continuous random variable.

In most practical engineering problems, continuous random variables represent measured data, such as temperature, time to failure, stress or strength. Discrete random variables represent count, or attribute data. Count data, such as the number of successes or failures, give no other information other than the fact that the device passed or failed. We are primarily concerned with continuous random variables.

2.2 Random Variables in Reliability

The primary continuous random variables which are significant to the evaluation of reliability include the following:

a) time-to-part failure (P) represents one part random variable

b) time-to the ith system failure (T_i) where i = 1, ..., n represents a set of n different system random variables

c) time-between the (i-1)st and ith system failures (S_i) where i = 1, ..., n represents a set of n individual system random variables

These random variables (P, T_i, S_i) are derived from the basic empirical failure data which is collected from both parts and systems. By observation of a, b and c above, it should be noted that empirical part data is described by a single random variable and the empirical data from a single system contains a sequence of (2 • n, where n is the total number of system failure events) primary random variables. It should also be noted that other system random variables could be defined, but for our purposes will not be considered as primary random variables. For example, a random variable could be specified which represented the time between first and fifth system failures.

These primary random variables are significant because of a very popular reliability modeling approach known as probabilistic modeling or distribution statistics (discussed in Section B). Probabilistic modeling is effective when applied to a <u>single</u> random variable whether it be a part or system random variable. Therefore, probabilistic modeling would be effective for each of the random variables identified in a-c above. In "a" above, a sample of identical parts would be required to apply probability modeling. In b and c, a sample of identical systems would be required to perform probability modeling on the resulting failure data. From this discussion, it has been identified that probability modeling is applied to a <u>single</u> random variable. It is also important to identify when probability modeling <u>should not</u> be applied. Probability modeling should not be applied to model the following because each represents a data set composed of multiple random variables:

a) interarrival times (TBF_i) from a single system failure process

b) arrival times ($TTSF_i$) from a single system failure process

c) mixed interarrival times from a group of identical systems

d) mixed arrival times from a group of identical systems

e) failure data from a group of different parts

So, when a data set can be characterized from (a) through (e) above, probability modeling techniques are invalid and should not be applied. Further insight as to why probability modeling techniques are not appropriate for these types of data sets is provided in Section 4.1.

2.3 Describing a Random Variable Using the Histogram

In reliability, we are usually most concerned with the possibility of success or failure of a system or part, and with the prediction of such numerics. Often times, this must be done using empirical data that was observed in a fielded application or a development test program. To draw inferences concerning the probability of failure based on this sample data, we must first determine how the random variable of interest is distributed. One way of making such determinations when sample failure data is available is through the use of a histogram.

The histogram graphically describes the frequency distribution of continuous random variables. A histogram is constructed by first establishing a series of intervals or cells over the sample range of the random variable. A recommended number of cells (k) in the histogram can be estimated by using Sturge's rule[1] :

$$k = 1 + 3.3 \log_{10} n \qquad (2\text{-}1)$$

where,

n = sample size

As an example, consider the data in Table 2.3-1, which represents the lifetimes of 40 similar car batteries recorded to the nearest tenth of a year. The car batteries are considered complex parts as discussed earlier in Section 1.3 and are discarded upon failure.

TABLE 2.3-1: CAR BATTERY LIFETIMES (YEARS)

2.2	4.0	3.6	4.4	3.1	3.8	2.9	2.7
3.4	1.7	3.0	3.4	3.8	3.0	4.6	3.7
2.5	4.2	3.5	3.5	2.8	3.4	3.8	3.2
3.2	3.2	3.6	4.5	3.2	4.0	2.0	3.3
4.8	3.7	3.3	2.5	3.8	3.1	4.1	3.6

1 H.A. Sturges, "The Choice of a Class Interval," ASA, 21, 65-66 (1926).

The cell interval for the above data set would be calculated using Sturge's rule as follows:

$k = 1 + 3.3 \log_{10} n$

$k = 1 + 3.3 \log_{10} 40$

$k = 6.3$ or 7 (when rounded up to nearest whole number)

Based on the Sturge's rule calculation, a total of seven cells will be used to subdivide the range of the sample data set given in Table 2.3-1.

Next, the cell width is calculated. The cell width is determined by dividing the range of the sample data by the number of recommended intervals. For our example, this is $(4.8 - 1.7)/7 = 0.443$. This represents the recommended cell width. Usually, we choose equal widths having the same number of significant digits as the observed data. Denoting this width by w, let's choose $w = 0.5$. Starting the first interval at 1.6, Table 2.3-2 shows the range and failure frequency (F) of each cell.

Next, the relative frequency (RF) is calculated by:

$$RF = \frac{\text{Number of failures during a cell interval}}{\text{Total number of items failed}} \tag{2-2}$$

The relative frequency is also shown on Table 2.3-2.

TABLE 2.3-2: RELATIVE FREQUENCY DISTRIBUTION OF BATTERY LIVES

Cell Interval	Cell Midpoint	Frequency (F)	Relative Frequency (RF)
$1.6 \leq x < 2.1$	1.85	2	0.050
$2.1 \leq x < 2.6$	2.35	3	0.075
$2.6 \leq x < 3.1$	2.85	5	0.125
$3.1 \leq x < 3.6$	3.35	13	0.325
$3.6 \leq x < 4.1$	3.85	11	0.275
$4.1 \leq x < 4.6$	4.35	4	0.100
$4.6 \leq x < 5.1$	4.85	2	0.050

Note: Battery Lives (years)

Finally, with the information provided in Table 2.3-2, a relative frequency histogram (also called a proportionate frequency histogram) can be constructed. The relative frequency histogram graphs the random variable by cell interval along the abscissa and relative frequency along the ordinate. The relative frequency histogram for the battery life data given in Table 2.3-1 is shown in Figure 2.3-1.

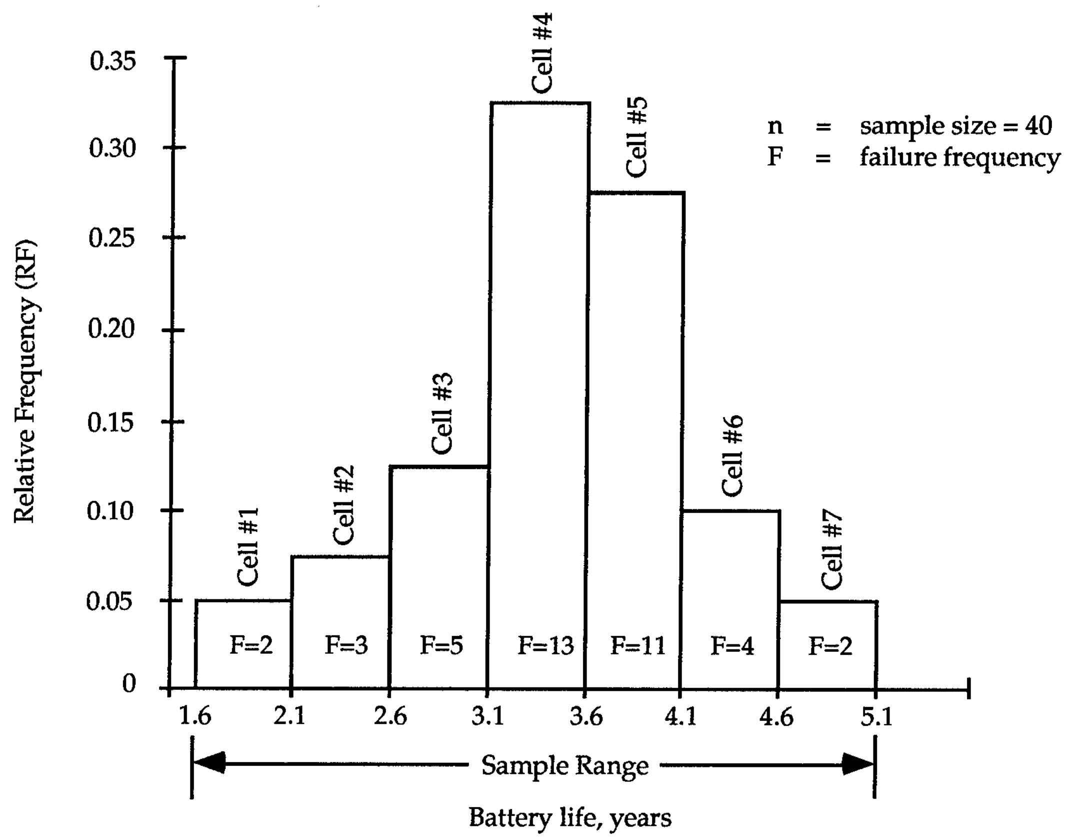


FIGURE 2.3-1: RELATIVE FREQUENCY HISTOGRAM FOR A SAMPLE OF BATTERIES

In Figure 2.3-1, the bars are drawn to have equal width, and are centered at the midpoint of each class interval. The height of each bar is equal to the observed relative frequency and represents a proportionate frequency or probability that a value of the random variable will occur within that interval.

2.4 Probability Density Function, f(x)

The relative frequency histogram discussed in Section 2.3 is an approximation, based on a limited sample size, of the probability distribution of a random variable. Stated more precisely, the limiting form of a relative frequency histogram, as the sample size approaches infinity, is the probability distribution.

This is represented graphically in Figure 2.4-1. As the sample size is increased, the histogram approaches a smooth curve, and the resulting curve is now a function of the random variable X, denoted by $f_X(x)$ and termed the probability density function. In general, a capital letter is used to represent the random variable and a lower case letter is used to represent a specific value of the random variable. In many instances, the random variable subscript (X) will be assumed and the probability density function will be represented as just f(x).

When f(x) represents a probability density function for a continuous random variable X, the expression f(x)dx (a measure of area) can be defined as the probability that values of the random variable fall between [x - (1/2)dx] and [x + (1/2)dx]. In general, the area under the probability density function represents probability as shown in Figure 2.4-2.

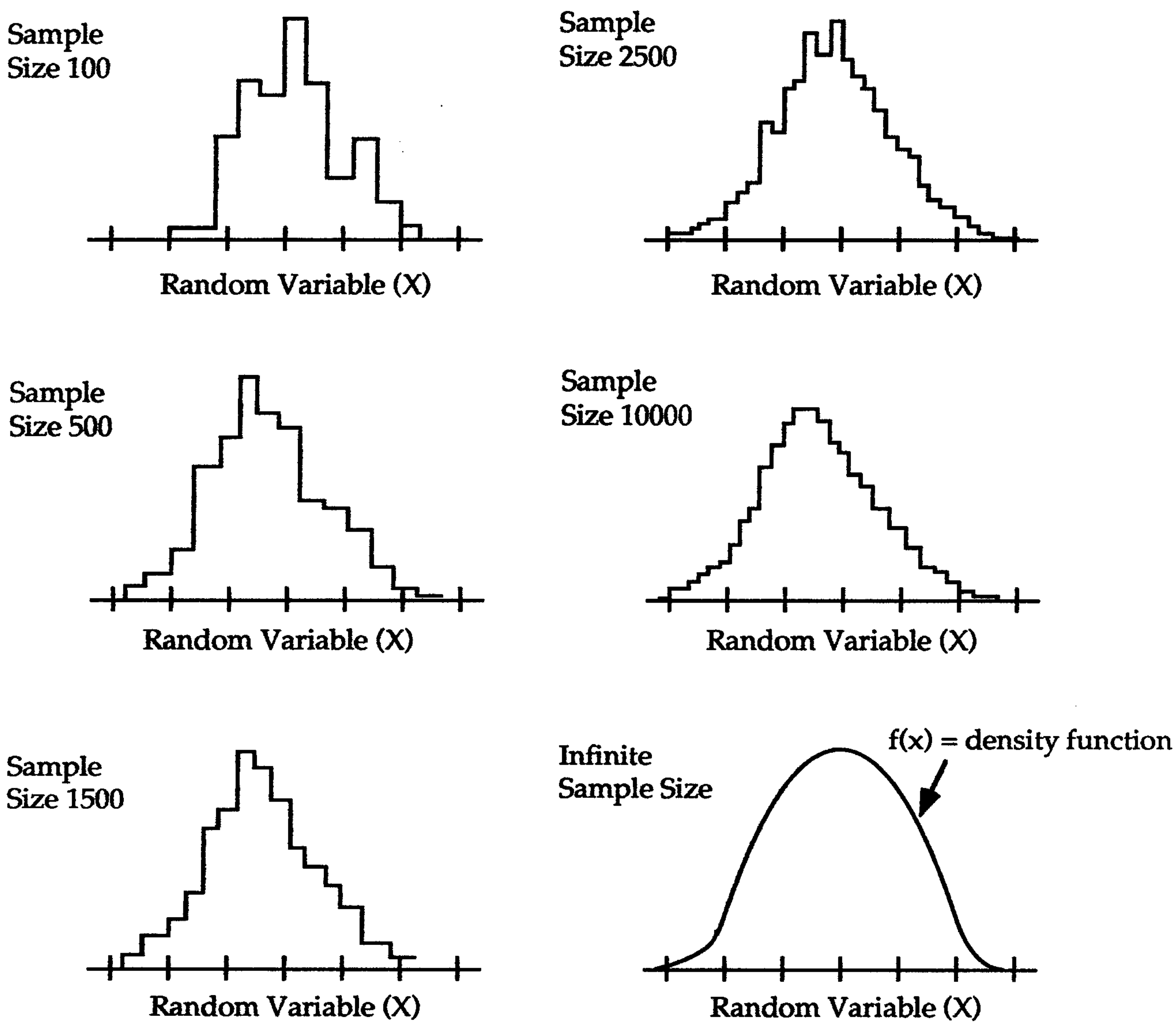


FIGURE 2.4-1: EFFECT OF INCREASED SAMPLE SIZE (n) ON THE RESOLUTION OF THE HISTOGRAM

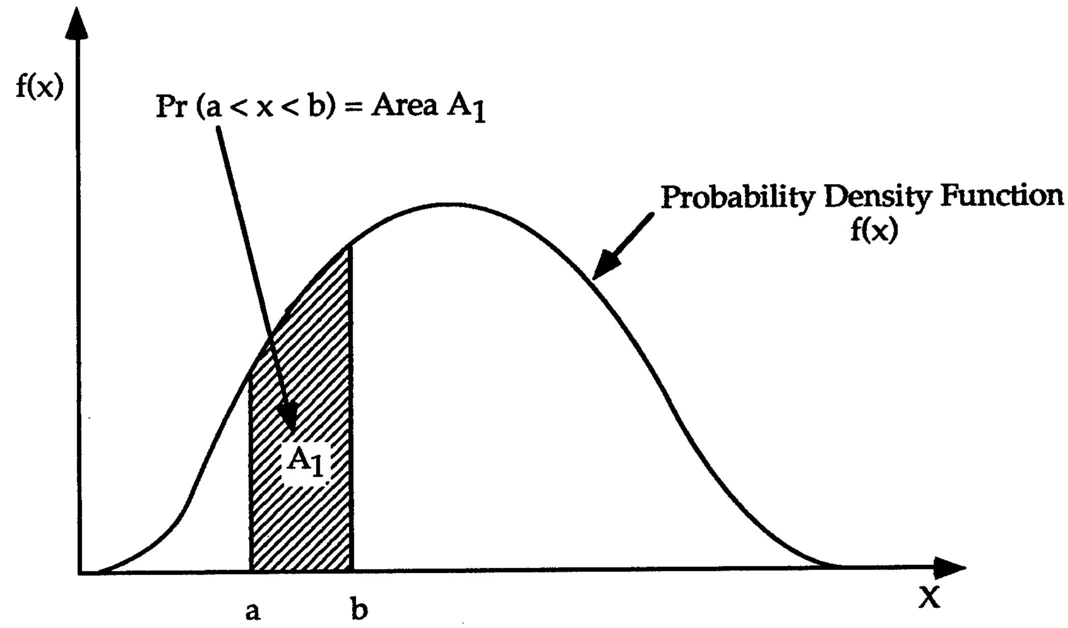


FIGURE 2.4-2: PROBABILITY OF AN EVENT OCCURRING DURING A SPECIFIED INTERVAL: a TO b

The probability that x (a specific value of random variable X) lies in some finite range, a to b, is generally represented as:

$$\Pr(a < x < b) = \int_a^b f(x)dx \tag{2-3}$$

Returning to the battery example, we can estimate the probability density function, f(x), by a smooth curve fitted to the relative frequency histogram as shown in Figure 2.4-3. The probability that a battery fails between 3.6 and 4.6 years when selected at random from an infinite line of production of such batteries can be determined by calculating the area under f(x) from 3.6 to 4.6. Mathematically, this is represented as:

$$\Pr(3.6 < x < 4.6) = \int_{3.6}^{4.6} f(x)dx \tag{2-4}$$

When the bounds of Equation (2-3) represent the extreme limits of all possible values of x, a characteristic of the probability density function is obtained; namely, the total area under the probability density function is equal to one or:

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{2-5}$$

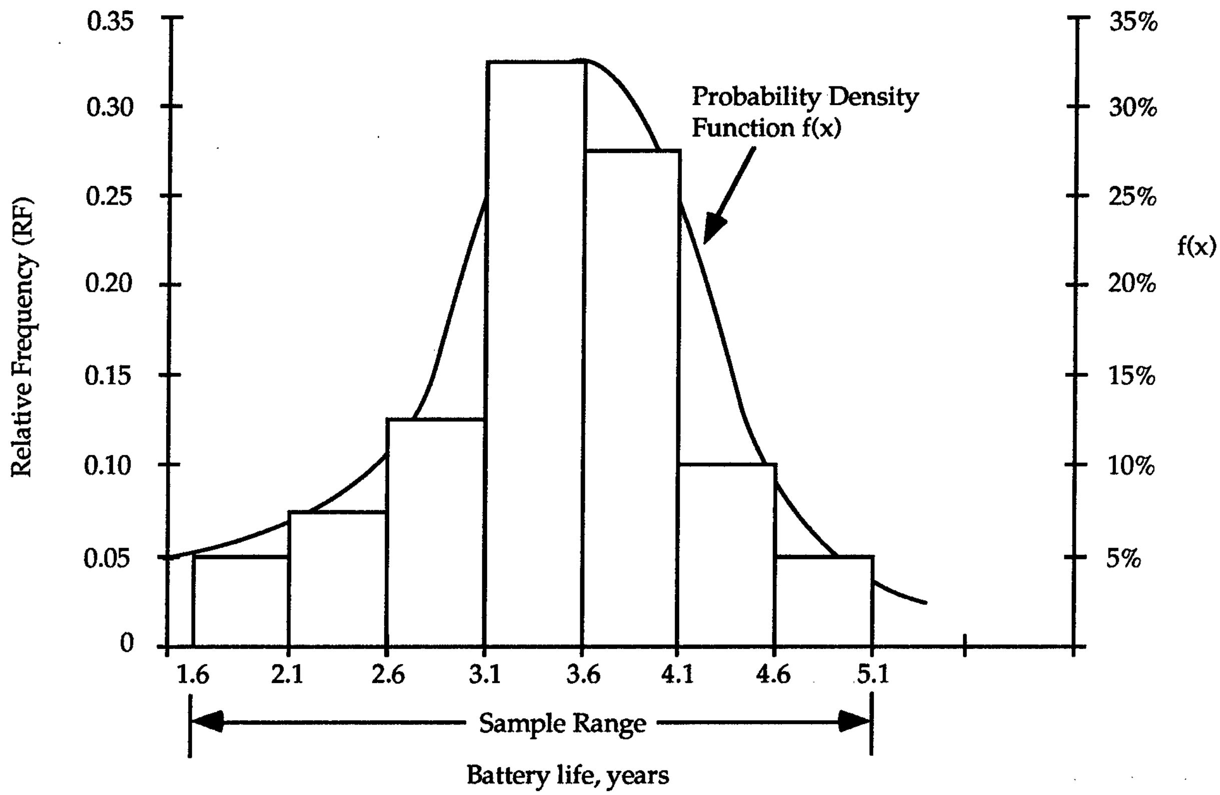


FIGURE 2.4-3: FITTING THE PROBABILITY DENSITY FUNCTION, f(x), TO THE RELATIVE FREQUENCY HISTOGRAM

Since probability is a dimensionless number that ranges from 0 to 1, one can now rationalize why f(x) is described as a probability function.

All probability density functions used as failure models in reliability are completely described by a random variable with range 0 to ∞. This premise is based on the rationale that life cannot be less than 0. For these density functions, Equation (2-5) can be represented as:

$$\int_0^\infty f(x)dx = 1 \tag{2-6}$$

Some of the more popular continuous probability distributions, defined by probability density functions, are listed in Table 2.4-1. The random variable range is also defined in Table 2.4-1 for each density function. Each of these probability distributions will be covered in greater detail later in Section 2.9.

TABLE 2.4-1: POPULAR CONTINUOUS PROBABILITY DISTRIBUTIONS

Probability Distribution	Probability Density Function, f(x)	Variate Range, x
Exponential	$f(x) = \lambda \exp(-\lambda x)$	$0 \leq x \leq \infty$
Weibull*	$f(x) = \frac{\beta}{\alpha^\beta}(x - x_0)^{\beta-1} \exp\left[-\left(\frac{x - x_0}{\alpha}\right)^\beta\right]$	$0 \leq x \leq \infty$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{(x - \mu)^2}{2\sigma^2}\right)\right]$	$-\infty \leq x \leq \infty$
Log-Normal	$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[-\left(\frac{(\ln x - \mu)^2}{2\sigma^2}\right)\right]$	$0 \leq x \leq \infty$

*Note, other mathematically equivalent forms are also available.

2.5 Relative Cumulative Frequency Polygon

The relative cumulative frequency polygon (also called the proportionate cumulative frequency polygon) is an important way of representing cumulative frequency data. The relative cumulative frequency polygon is obtained by plotting the random variable on the abscissa and the relative cumulative frequency on the ordinate. The cumulative plot is useful for reading various values at a glance. For instance, what percent of batteries from the example in Section 2.3 will fail during the first 3.6 years? To determine this, we can plot the relative cumulative frequency polygon of X (the life of the car battery). The relative cumulative frequency data is summarized in Table 2.5-1, being derived from the original data of Table 2.3-1. Next, we plot the relative cumulative frequency against the upper cell boundary as shown in Figure 2.5-1. We can now quickly see from Figure 2.5-1 that 57% of the batteries will fail during the first 3.6 years.

TABLE 2.5-1: RELATIVE CUMULATIVE FREQUENCY DATA

Cell Boundary	Cumulative Failure Frequency	Relative Cumulative Frequency
x < 1.6	0	.0
x < 2.1	2	.050
x < 2.6	5	.125
x < 3.1	10	.250
x < 3.6	23	.575
x < 4.1	34	.850
x < 4.6	38	.950
x < 5.1	40	1.00

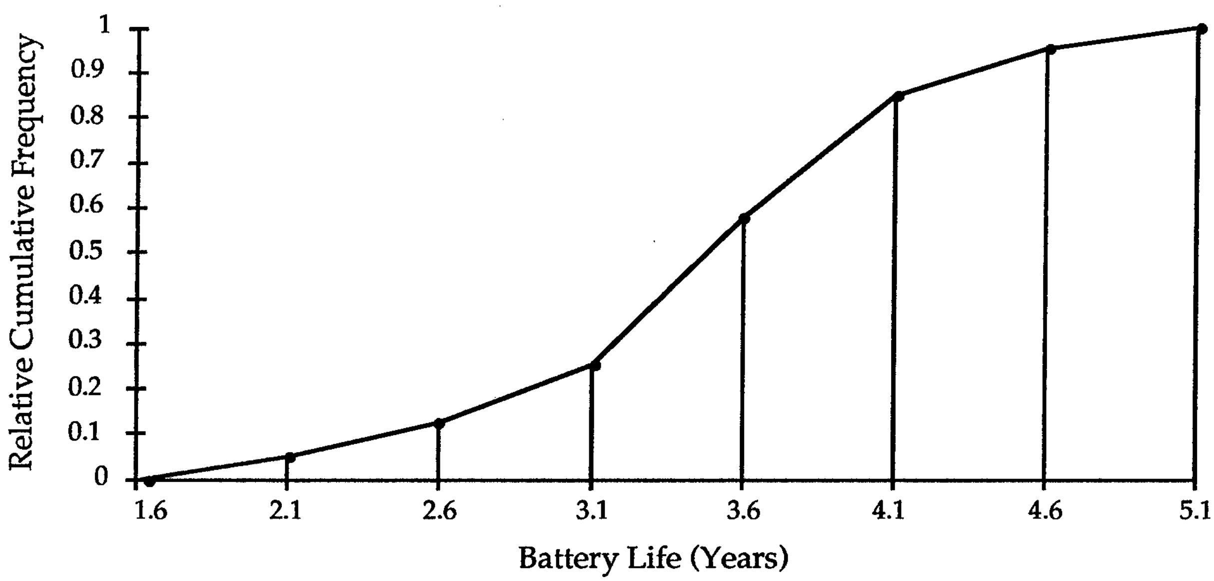


FIGURE 2.5-1: RELATIVE CUMULATIVE FREQUENCY POLYGON

From the relative cumulative frequency polygon, we can estimate the cumulative distribution function, F(x), by fitting a smooth curve to the data points as shown in Figure 2.5-2. As the sample size approaches infinity, the limiting form of the relative cumulative frequency polygon is the cumulative distribution function F(x). The cumulative distribution function, F(x), will be considered in more detail in Section 2.6.

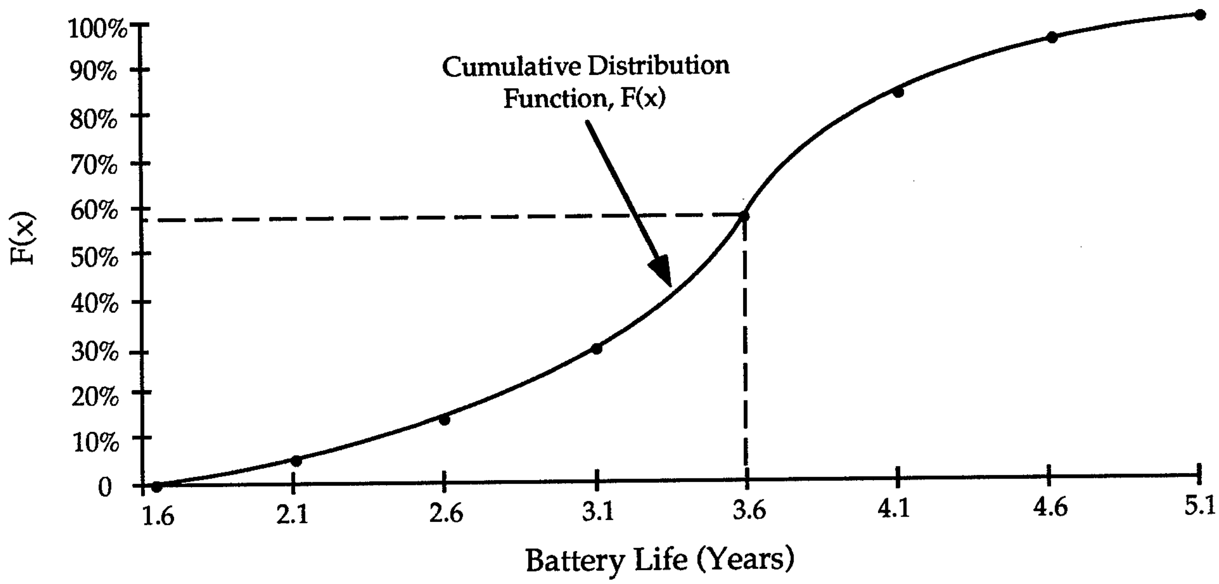


FIGURE 2.5-2: CUMULATIVE DISTRIBUTION FUNCTION, F(x)

2.6 Cumulative Distribution Function, F(x)

The cumulative distribution function, F(x), of a continuous random variable X, having a probability density function, f(x), is given as:

$$F(x) = \Pr(x \le a) = \int_{-\infty}^{a} f(x)dx \tag{2-7}$$

The cumulative distribution function is typically used to determine the probability that a random variable is <u>not</u> greater than a specified value.

A more particular form of Equation (2-7) arises when the range considered is from the lower limit of the random variable, which is often zero (refer to Table 2.4-1), to some specific value, a, of the random variable. Thus, the cumulative distribution function, F(x), or the probability that the random variable <u>is not</u> greater than a specific value, a, is:

$$\Pr(0 \le x \le a) = \int_{0}^{a} f(x)dx \tag{2-8}$$

The cumulative distribution function F(x) can then be expressed as:

$$F(x) = \int_{0}^{x} f(x)dx \tag{2-9}$$

When the random variable is time-to-failure or life, Equation (2-9) represents the probability of failure prior to some time x as shown in Figure 2.6-1.

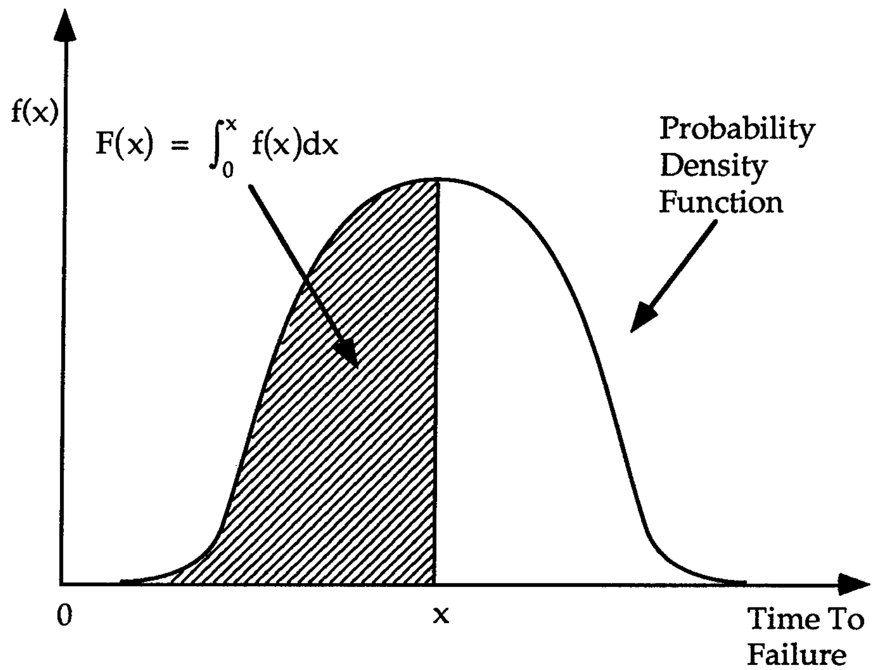


FIGURE 2.6-1: PROBABILITY OF FAILURE AS REPRESENTED BY THE AREA UNDER THE PROBABILITY DENSITY FUNCTION

The cumulative distribution function can also be used to evaluate the probability of an event occurring in a specified range, $a < x < b$, by:

$$Pr(a < x < b) = F(b) - F(a) \tag{2-10}$$

Furthermore, the derivative of the cumulative distribution function is the probability density function, or:

$$f(x) = \frac{dF(x)}{dx} \tag{2-11}$$

A simple example is now presented to show the utility of the probability density function and cumulative distribution function (Reference [3]).

Example A. Given the following probability density function:

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

1. Verify that the total area under the curve is 1.

2. Find Pr (0 < x < 2)

<u>Solution A</u>:

1. $$\int_{-\infty}^{\infty} f(x)dx = \int_{-1}^{2} \frac{x^2}{3} dx = \frac{x^3}{9}\bigg|_{-1}^{2} = \frac{8}{9} + \frac{1}{9} = 1$$

2. $$\Pr(0 < x < 2) = \int_{0}^{2} \frac{x^2}{3} dx = \frac{x^3}{9}\bigg|_{0}^{2} = \frac{8}{9} \text{ or } 89\%$$

Example B. For the probability density function above, find F(x) and evaluate Pr (0 < x < 2)

<u>Solution B</u>:

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-1}^{x} \frac{x^2}{3} = \frac{x^3}{9}\bigg|_{-1}^{x} = \frac{x^3 + 1}{9}$$

$$\Pr(0 < x < 2) = F(2) - F(0) = \frac{9}{9} - \frac{1}{9} = \frac{8}{9}$$

which agrees with solution A above.

2.7 Survivor or Reliability Function, R(x)

The reliability function R(x) which is defined as the probability of a device not failing prior to some time, x, is given by:

$$R(x) = 1 - F(x) = 1 - \int_0^x f(x)dx \tag{2-12}$$

or

$$R(x) = \int_x^\infty f(x)dx \tag{2-13}$$

In other words, R(x) is the probability of survival at time x given failure has not occurred prior to x.

The application of the survivor function, R(x), to describe the reliability of a part population can be done simply by substituting the appropriate density function (i.e., failure model) into Equation (2-13). The procedure becomes conceptually more difficult when we require a reliability statement for a system since a system exhibits a sequence of failures.

The question arises of how to postulate a survivor function for a system. This problem will be discussed in more detail in Section D. But for now, it will help to interpret the survivor function of Equation (2-13) as:

$$R(0, x) = \int_x^\infty f(x)dx \tag{2-14}$$

This improves our interpretation of the survivor function or reliability function to emphasize that reliability is defined over a specified time interval. In the case of parts, reliability is defined as probability of no failure in the interval 0 to some time x.

The characteristics of the reliability function are: (1) at time zero the reliability function is one and (2) as time approaches infinity, the reliability function approaches zero. These characteristics are illustrated in Figure 2.7-1. The reliability expressed in Equation (2-13) is illustrated in Figure 2.7-2.

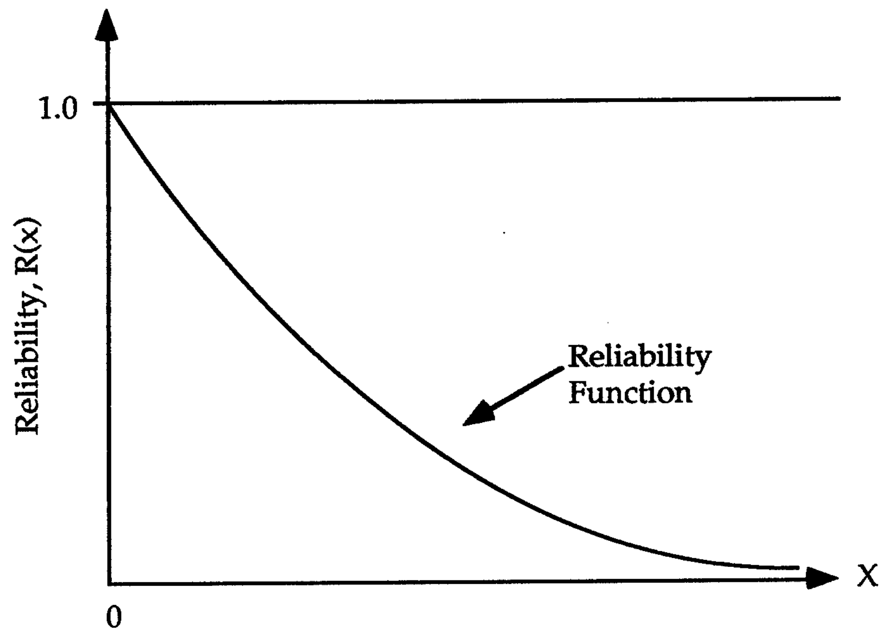


FIGURE 2.7-1: GENERAL RELIABILITY FUNCTION

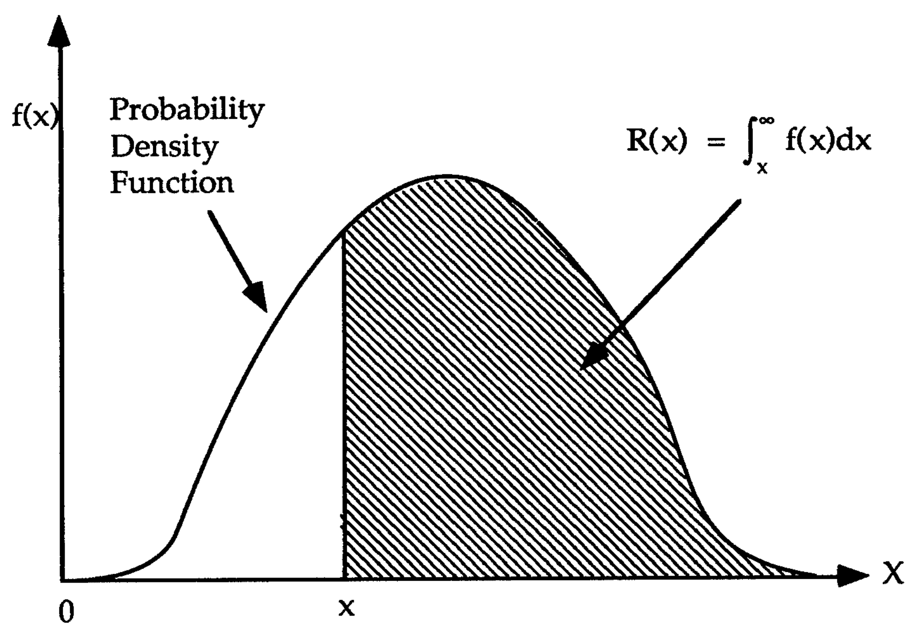


FIGURE 2.7-2: RELIABILITY REPRESENTED BY THE AREA UNDER THE PROBABILITY DENSITY FUNCTION

Differentiation of Equation (2-12) yields:

$$\frac{dR(x)}{dx} = -\frac{dF(x)}{dx} = -f(x) \tag{2-15}$$

Equation (2-15) will be used in Section 2.8 to derive the hazard rate.

2.8 Hazard Rate, h(x)

The hazard rate, h(x), or the force of mortality (FOM) is a conditional expression that gives the probability that a device already in service for time x will fail in the next instant of time, dx, given that it has not failed previously. Thus, it essentially shows how the risk of failure changes over time.

Let F(x) be the cumulative distribution function of the time-to-failure random variable X, and let f(x) be its probability density function. Then the hazard function, h(x), can be derived as follows:

The probability of failure in a given time interval between x and x + Δx can be expressed as:

$$\int_{x}^{\infty} f(x)dx - \int_{x+\Delta x}^{\infty} f(x)dx = R(x) - R(x + \Delta x) \tag{2-16}$$

where R(x) is the reliability or probability of survival at time x.

The risk of failure, λ(x), in the interval x to x + Δx is defined as the probability that failure occurs in the interval (Equation (2-16)), divided by the product of the probability that failure does not occur prior to the start of the interval (Equation (2-13)) and the interval length, or

$$\lambda(x) = \frac{R(x) - R(x + \Delta x)}{R(x)\,\Delta x} \tag{2-17}$$

The hazard rate, h(x), or force of motality is defined as the limit of λ(x) as the interval length approaches zero, or

$$h(x) = \lim_{\Delta x \to 0} \left[\frac{R(x) - R(x + \Delta x)}{R(x)\, \Delta x} \right] \tag{2-18}$$

or

$$h(x) = \frac{1}{R(x)} \lim_{\Delta x \to 0} \left[\frac{R(x) - R(x + \Delta x)}{\Delta x} \right] \tag{2-19}$$

Using the definition of a derivative, Equation (2-19) becomes:

$$h(x) = \frac{1}{R(x)} \left[\frac{-d\, R(x)}{dx} \right] \tag{2-20}$$

It was shown previously in Equation (2-15) that

$$f(x) = \frac{-d\, R(x)}{dx} \tag{2-21}$$

Substitution of Equation (2-21) into Equation (2-20) yields the definition of the hazard function:

$$h(x) = \frac{f(x)}{R(x)} = \frac{f(x)}{1 - F(x)} \tag{2-22}$$

The hazard function is one of the fundamental relationships important in reliability analysis. It is important to note that even though the hazard rate is defined by probability functions, the hazard rate is <u>not</u> a probability function.

The reliability function, R(x), can be derived strictly in terms of the hazard rate function, h(x), as follows.

Equation (2-22) is expressed as:

$$f(x) = h(x)\,[1 - F(x)] \tag{2-23}$$

The cumulative distribution function was previously defined as:

$$F(x) = \int_0^x f(x)dx \tag{2-24}$$

The derivative of the cumulative distribution function is:

$$\frac{d\,F(x)}{dx} = f(x) \tag{2-25}$$

Substituting Equation (2-23) into Equation (2-25) yields:

$$\frac{dF(x)}{dx} = h(x)\,[1 - F(x)] \tag{2-26}$$

or,

$$h(x)dx = \frac{d\,F(x)}{1 - F(x)} \tag{2-27}$$

Integrating Equation (2-27) yields:

$$\int_0^x h(x)dx = \int_0^x \frac{dF(x)}{1 - F(x)} \tag{2-28}$$

$$\int_0^x h(x)dx = -\ln\,[1 - F(x)]\,\Big|_0^x \tag{2-29}$$

$$\int_0^x h(x)dx = -\ln\,[1 - F(x)] + \ln\,[1 - F(0)] \tag{2-30}$$

Since F(0) = 0, Equation (2-30) reduces to:

$$\int_0^x h(x)dx = -\ln\,[1 - F(x)] \tag{2-31}$$

The reliability function, R(x), expressed in terms of the hazard function is then expressed as:

$$R(x) = 1 - F(x) = \exp\left[-\int_0^x h(x)dx\right] \tag{2-32}$$

Using Equations (2-32) and (2-22) the probability density function, f(x), can also be expressed entirely in terms of the hazard function.

$$f(x) = h(x)\exp\left[-\int_0^x h(x)dx\right] \tag{2-33}$$

A summary of the important relationships involving the hazard function is provided in Table 2.8-1.

TABLE 2.8-1: SUMMARY OF IMPORTANT RELATIONSHIPS INVOLVING THE HAZARD FUNCTION

Relationship	Interpretation
$h(x) = \dfrac{f(x)}{R(x)} = \dfrac{f(x)}{1-F(x)}$	Hazard rate in terms of the survivor function, R(x), and the cumulative distribution function, F(x)
$R(x) = 1-F(x) = \exp\left[-\int_0^x h(x)dx\right]$	Survivor function, R(x), expressed in terms of the hazard rate
$f(x) = h(x)\exp\left[-\int_0^x h(x)dx\right]$	Probability density function, f(x), expressed in terms of the hazard rate

2.9 Probability Distributions for Continuous Random Variables

Although there are several probability distributions to choose from, experience has shown that for reliability work, a relatively small number of the distributions will satisfy most needs. This section provides a summary of the following distributions most commonly used for reliability work.

1) Exponential
2) Weibull
3) Normal
4) Log-normal
5) Extreme-value

2.9.1 Exponential Distribution

The exponential distribution is defined by the following probability density function:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x > 0, \lambda > 0 \\ 0 & , \text{elsewhere} \end{cases} \qquad (2\text{-}34)$$

The cumulative distribution function is defined to be:

$$F(x) = \int_{-\infty}^{x} f(x)dx = \int_{0}^{x} \lambda e^{-\lambda x}\, dx = -e^{-\lambda x}\Big|_{0}^{x}$$

or,

$$F(x) = 1 - e^{-\lambda x} \tag{2-35}$$

In the past, the exponential distribution has been the most commonly used distribution in reliability to model the failure characteristics of parts. However, the widespread use of the exponential distribution is not necessarily an indication that it is always appropriate. In fact, for mechanical parts, it is usually inappropriate. The reason for its popularity lies in the mathematical simplicity of the resulting functional expressions for reliability and the hazard rate.

The reliability function for the exponential distribution is defined as:

$$R(x) = 1 - F(x) = 1 - \int_{0}^{x} f(x)dt \tag{2-36}$$

$$R(x) = e^{-\lambda x} \tag{2-37}$$

Recall that the hazard rate is defined as:

$$h(x) = \frac{f(x)}{R(x)} \tag{2-38}$$

Substituting Equations (2-37) and (2-34) into Equation (2-38) yields:

$$h(x) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

$$h(x) = \lambda \text{, (a constant for the exponential case)} \tag{2-39}$$

When time-to-failure is the random variable of interest, the parameter λ has become known as the "failure rate". Therefore, for the exponential case, the hazard rate is constant over time. This means that regardless of how many hours a device has survived (100, 500 or 1000 hours) the risk of failure in the next instant is the same. A constant hazard rate also characterizes "random" failures.

The simplicity of the Equations (2-37) and (2-39) has been a contributing factor to the overwhelming popularity of the exponential distribution. Another contributing factor has been the lack of sufficient time-to-failure data that could be used to better characterize other failure distributions. These two factors have led many engineers to assume a constant hazard rate and an exponential failure model.

2.9.2 Weibull Distribution

A distribution that continues to gain popularity in reliability work is the Weibull distribution. The original work regarding this distribution was presented in a hallmark paper which appeared in the Journal of Applied Mechanics in 1951[2]. As the paper had indicated, it has indeed become "a statistical distribution function of wide applicability". The reason for its growth in popularity is its versatility. Many of the distributions used in reliability and described in Section 2.9, can be derived from or approximated by the Weibull density function which is defined as:

$$f(x) = \begin{cases} \left[\frac{\beta}{\alpha^\beta}(x - x_o)^{\beta-1}\right] \exp\left[-\left(\frac{x - x_o}{\alpha}\right)^\beta\right], x \geq x_o \\ 0, \text{Elsewhere} \end{cases} \tag{2-40}$$

where,

x_o = expected minimum value of the random variable
β = shape parameter or Weibull slope ($\beta > 0$)
α = scale parameter or characteristic value ($\alpha > 0$)

The cumulative distribution function for the Weibull distribution can be derived by substituting Equation (2-40) into Equation (2-9), the result is:

$$F(x) = 1 - \exp\left[-\left(\frac{x - x_o}{\alpha}\right)^\beta\right], x \geq x_o \tag{2-41}$$

2 Weibull, Waloddi (1951). "A Statistical Distribution Function of Wide Applicability", Journal of Applied Mechanics, pg. 293-297.

As noted, several of the other distributions can be derived from the Weibull distribution. Modeling other distributions can be accomplished by selecting the appropriate value of β, the Weibull shape parameter. Table 2.9-1 provides a list of distributions and their corresponding values of β.

TABLE 2.9-1: WEIBULL SHAPE PARAMETER, β, AND RESULTING DISTRIBUTION

Shape Parameter Value	Corresponding Distribution
$\beta < 1$	Gamma (k<1)
$\beta = 1$	Exponential
$\beta = 2$	Raleigh
$\beta = 1.5$	Log-normal (approximate)
$\beta = 3.44$	Normal (approximate)

Equation (2-40) represents a three parameter Weibull distribution. In the study of reliability, where it is reasonable to assume that a lower bound on life is zero, the variable x_0 in Equation (2-40) can be set to zero and a two parameter Weibull distribution is created, where,

$$f(x) = \begin{cases} \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right], x > 0, \alpha > 0, \beta > 0 \\ 0, \text{Elsewhere} \end{cases} \tag{2-42}$$

and

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \tag{2-43}$$

The characteristic value, α, can be expressed in terms of the cumulative distribution function by setting $x = \alpha$, then:

$$F(\alpha) = 1 - \exp\left[-\left(\frac{\alpha}{\alpha}\right)^{\beta}\right] \tag{2-44}$$

$$F(\alpha) = 1 - \exp(-1)$$

$F(\alpha) = .632$ of 63.2%

Therefore, the characteristic value, α, is that value at which 63.2% of the population will have failed when the random variable considered is life.

Using Equations (2-40) and (2-13), we can derive the reliability function for the three parameter Weibull distribution as,

$$R(x) = \exp\left[-\left(\frac{x-x_0}{\alpha}\right)^{\beta}\right] \tag{2-45}$$

and the Weibull hazard rate is derived from Equation (2-22) as:

$$h(x) = \frac{\beta}{\alpha}\left(\frac{x-x_0}{\alpha}\right)^{\beta-1} = \frac{\beta}{\alpha^{\beta}}(x-x_0)^{\beta-1} \tag{2-46}$$

The Weibull probability density function and the hazard rate are sketched for five values of the shape parameter, β, in Figure 2.9-1 and 2.9-2, respectively. Figure 2.9-2 indicates that the Weibull hazard function is an increasing function when $\beta > 1$ and is independent of the random variable when $\beta=1$. When $\beta < 1$ the hazard rate decreases as the random variable increases. This illustrates the versatility of the Weibull distribution to represent a family of various distributions.

Many mechanical components are characterized by an increasing hazard rate ($\beta > 1$) due to deterioration or wear. A decreasing hazard rate ($\beta < 1$) is useful in characterizing phenomena such as work hardening and other life improvement processes. The constant hazard rate ($\beta=1$) is generally used to characterize random failures such as those exhibited by many electronic components. A detailed treatment of the utility of the Weibull distribution can be found in Reference [2].

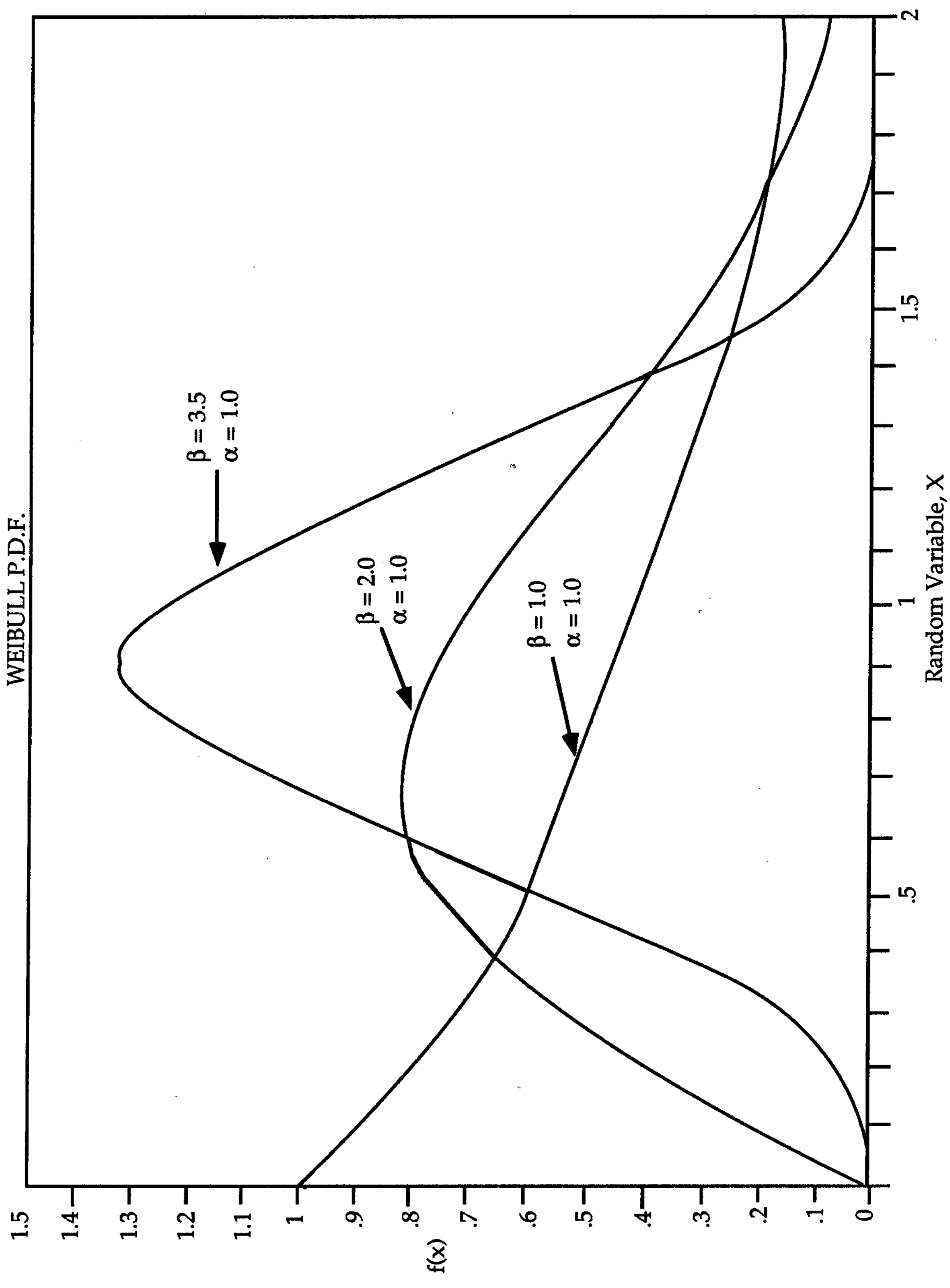


FIGURE 2.9-1: WEIBULL DENSITY FUNCTIONS, VARIOUS SHAPES (β)

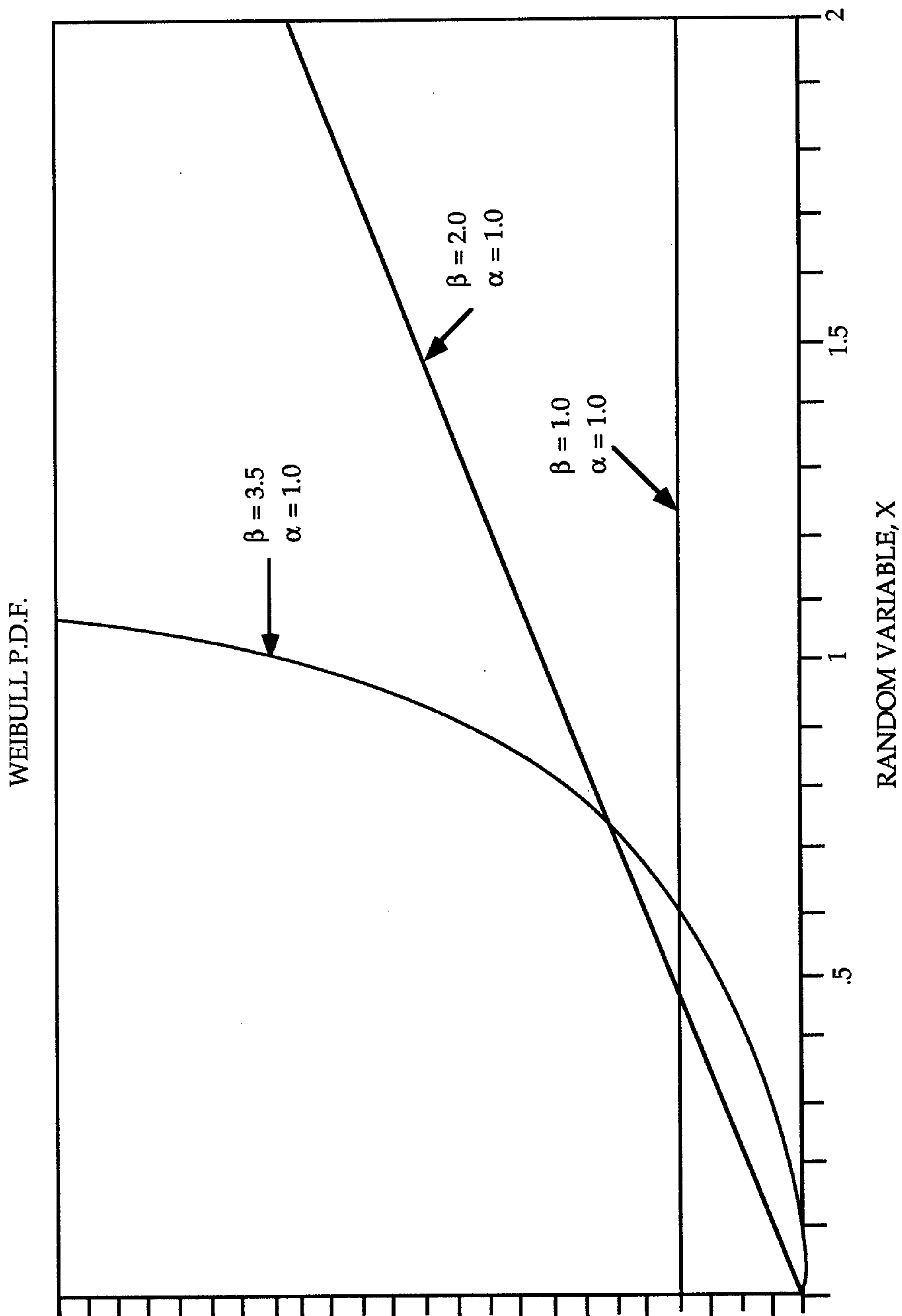


FIGURE 2.9-2: WEIBULL HAZARD FUNCTIONS, VARIOUS SHAPES (β)

2.9.3 Normal Distribution

The normal density function was derived by Carl F. Gauss in 1809 in his investigations of the mathematics of planetary orbits. Since that time, the normal distribution has become one of the best known and most widely used statistical distributions. Even though the normal distribution has limited use for analyzing failure data as discussed below, it is very useful as a model for characterizing other random variables used in reliability analyses. For example, the normal distribution is frequently used to describe the stress or strength of a part in the application of interference analysis (refer to Section 3.6). In general, the normal distribution is a good representative model for the distribution of variables from many naturally occurring phenomena which are expected to be symmetric.

The probability density function for the normal distribution is represented by the mathematical function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right], \; -\infty \le x \le \infty \tag{2-47}$$

where,

$f(x)$ = probability of occurrence of x
μ = measure of central tendency (mean of the population)
σ = standard deviation
σ^2 = variance

The normal distribution is not generally utilized as a failure model because it does not satisfy the conditions required for a time-to-failure probability density function. In real world situations, the time-to-failure for both parts and systems must be positive and the probability density function (i.e., failure model) is then distributed from 0 to ∞, or

$$\int_0^\infty f(x)dx = 1 \tag{2-48}$$

Instead, the normal distribution satisfies:

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{2-49}$$

This situation can be rectified by modifying the normal distribution so as to fully comply with the conditions for a time-to-failure distribution. The modification function has the form:

$$g(x) = \frac{f(x)}{1 - \int_{-\infty}^{0} f(x)dx}, \; 0 < x < \infty \tag{2-50}$$

However, g(x) is no longer a normal distribution.

As with the Weibull distribution, the normal distribution is representative of a family of distributions, each member with unique values of μ and σ^2 in the case of the normal distribution. A nice feature of the normal distribution is that any normal distribution can be transformed to a standard normal distribution with a mean (μ) of zero and a variance $\left(\sigma^2\right)$ of one. The standard normal density function can be derived from Equation (2-47) by setting $\mu = 0$ and $\sigma^2 = 1$.

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-x^2}{2}\right] \tag{2-51}$$

In order to transform any non-standard normal distribution to a standard normal distribution, the following transformation can be applied:

$$z = \frac{x - \mu}{\sigma} \tag{2-52}$$

where,

z = transformed value of x
μ = mean of x
σ = standard deviation of x

An application of Equations (2-51) and (2-52) will be presented in Section 3.6.2 of this document.

Another use of the normal distribution involves its application to system parameter tolerance analysis. A tolerance analysis is designed to look at the consequences on system outputs, due to the simultaneous "drift" of individual components. Several computer programs are available to aid in this kind of analysis, by simulating system behavior while choosing various values of component parameters from an assumed normal parameter distribution.

2.9.4 Log-Normal Distribution

Like the Weibull distribution, the log-normal distribution is a versatile statistical distribution which can assume a range of shapes. The log-normal does not have the normal distribution's disadvantage of the variate extending below zero to -∞. The log-normal distribution is often found to be a good fit to empirical data. Therefore, the log-normal distribution is a natural candidate as a time-to-failure model. Of late, the log-normal distribution has gained popularity because of its applicability to the reliability analysis of the fatigue life of certain types of mechanical components and to the maintainability analysis of time-to-repair data.

The log-normal distribution implies that the logarithms of the random variable are normally distributed. The log-normal density function is:

$$f(x) = \begin{cases} \dfrac{1}{\sigma x\,(2\pi)^{1/2}} \exp -\left[\dfrac{(\ln x-\mu)^2}{2\sigma^2}\right], & 0 < x < \infty \\ 0 & \text{, Elsewhere} \end{cases} \tag{2-53}$$

The mean and standard deviation of the log-normal distribution are given by:

$$\text{Mean} = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{2-54}$$

$$\text{Standard Deviation} = \left[\exp\left(2\mu + 2\sigma^2\right) - \exp\left(2\mu + \sigma^2\right)\right]^{\frac{1}{2}} \tag{2-55}$$

where μ and σ are the mean and standard deviation of the normal distribution whose variate is the natural logarithm of the data.

The hazard function for the log-normal distribution involves the standard normal probability density function, $\phi(x)$, and the standard normal cumulative distribution function, $\Phi(x)$. The hazard function is given by:

$$h(x) = \frac{\phi\left(\frac{\ln x - \mu}{\sigma}\right)}{\sigma x - \sigma x\,\Phi\left(\frac{\ln x - \mu}{\sigma}\right)} \tag{2-56}$$

The hazard functions for the log-normal distribution quickly increase to a maximum value then decrease relatively slowly.

The log-normal probability density function and the hazard functions are shown in Figure 2.9-3 and Figure 2.9-4, respectively, for various choices of μ and σ.

2.9.5 Type I Extreme-Value or Gumbel Distribution

Three types of extreme-value distributions were developed by Gumbel (1958) for describing minimum or maximum extreme values. Type II and III Extreme-Value distributions are not typically useful in failure studies. The Type I Extreme-Value distribution has found utility as a failure model especially in cases where failure is due to a corrosive process.

In reliability work, the failure of a component may frequently be linked to phenomena which occurs at the extremes. In these cases we are interested in the distribution of the minimum or maximum value in a sample from some initial distribution of the common population. It can not be assumed that the distribution of the population is necessarily a good model for the extremes. Extreme-value statistics were developed to describe these situations.

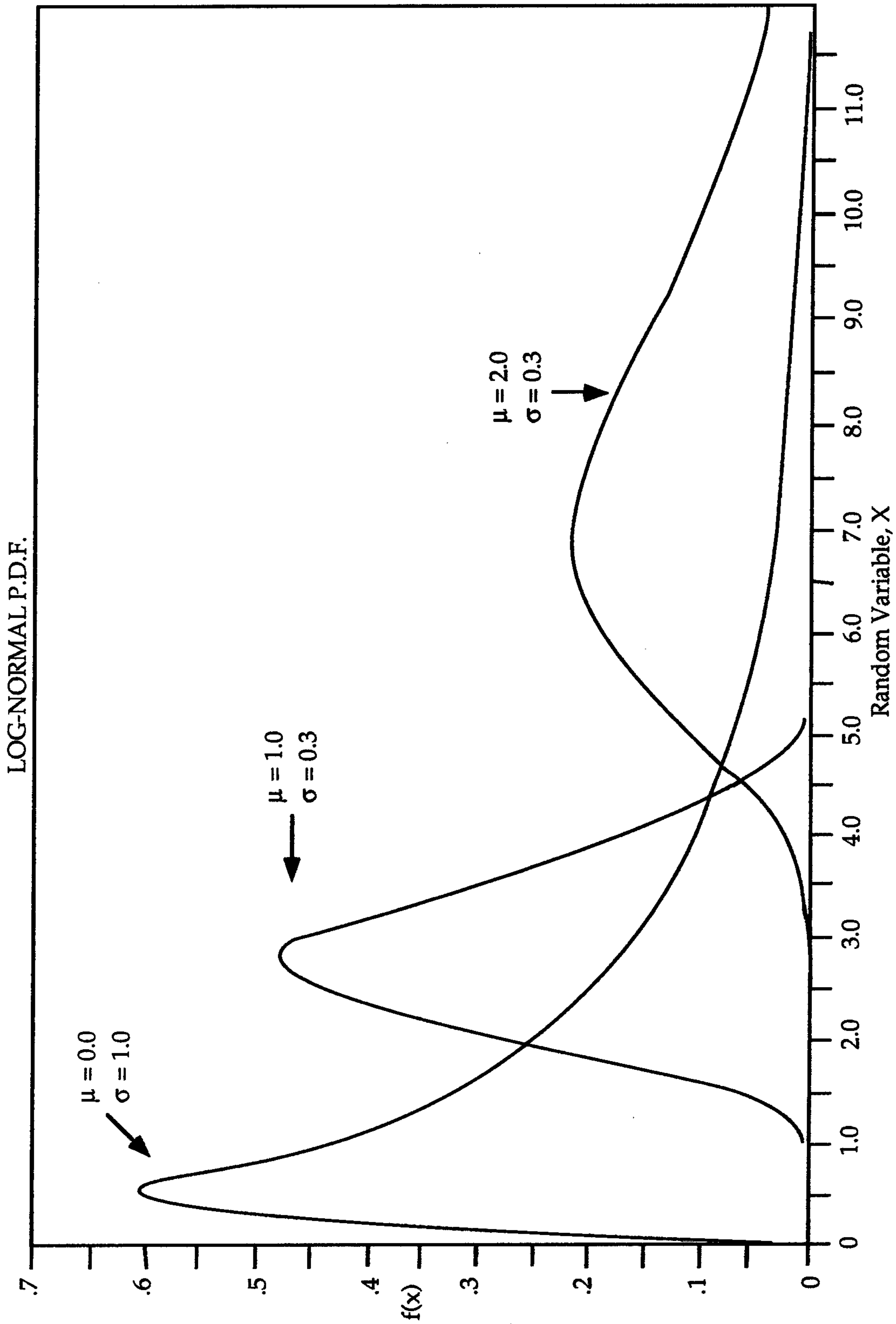


FIGURE 2.9-3: LOG-NORMAL DISTRIBUTION

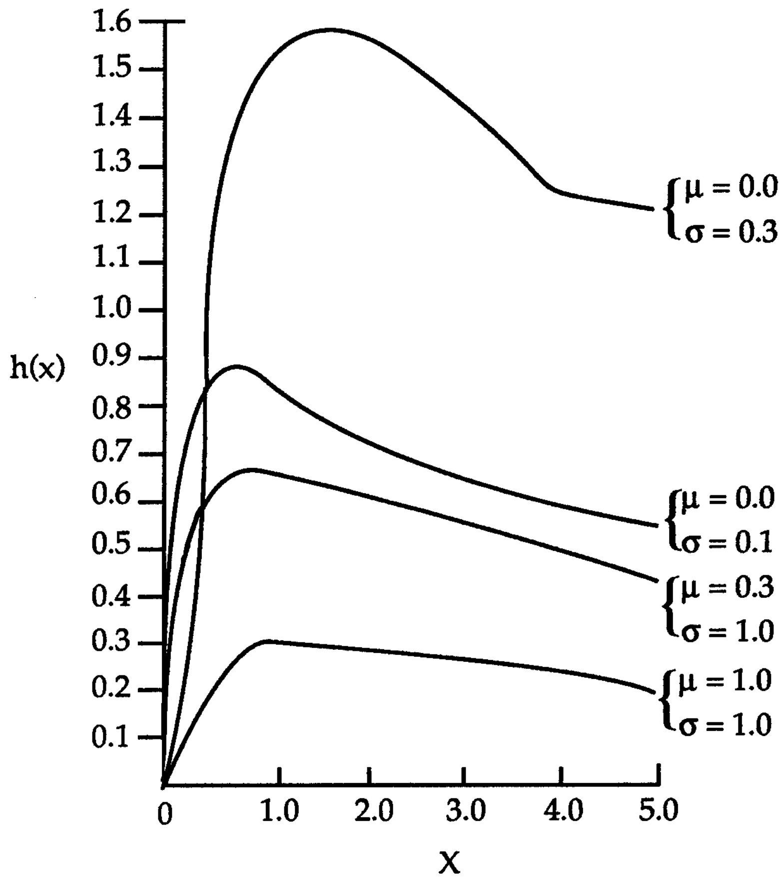


FIGURE 2.9-4: HAZARD FUNCTIONS FOR LOG-NORMAL DISTRIBUTIONS WITH DIFFERENT PARAMETER VALUES

The Type I Extreme-Value distribution for maximum or minimum values is applicable when the underlying distribution for the extremes is of the exponential type. These are distributions whose cumulative probability approaches unity at a rate which is equal to or greater than that for the exponential distribution. This includes most reliability distributions, such as the normal, log-normal and exponential distributions.

The probability density function, f(x), and the hazard function, h(x), for the Type I Extreme-Value distribution of <u>maximum elements</u> are:

$$f(x) = \frac{1}{\sigma} \exp\left\{-\frac{1}{\sigma}(x-\mu) - \exp\left[-(1/\sigma)(x-\mu)\right]\right\} \tag{2-57}$$

$$-\infty < x < \infty,\ -\infty < \mu < \infty,\ \sigma > 0$$

$$h(x) = \frac{\exp[-(1/\sigma)(x-\mu)]}{\sigma\{\exp[\exp(-(1/\sigma)(x-\mu))] - 1\}} \qquad (2\text{-}58)$$

The parameters μ and σ are the location and the scale parameters, respectively, of the distribution.

The probability density function, f(x), and the hazard function, h(x), for the Type I Extreme-Value distribution of minimum elements are:

$$f(x) = \frac{1}{\sigma}\exp\left\{\frac{1}{\sigma}(x - \mu) - \exp\left[(1/\sigma)(x - \mu)\right]\right\} \qquad (2\text{-}59)$$

$$-\infty < x < \infty,\ -\infty < \mu < \infty,\ \sigma > 0$$

$$h(x) = \frac{1}{\sigma}\exp\left(\frac{x - \mu}{\sigma}\right) \qquad (2\text{-}60)$$

Note that the Type I Extreme-Value probability density functions contain no shape parameter, and thus there is only a single shape. This limits the versatility of the Type I Extreme-Value distribution. Plots of the probability density functions are given in Figure 2.9-5 when $\mu = 5$ and $\sigma = 1$. Graphs of the hazard functions are given in Figure 2.9-6.

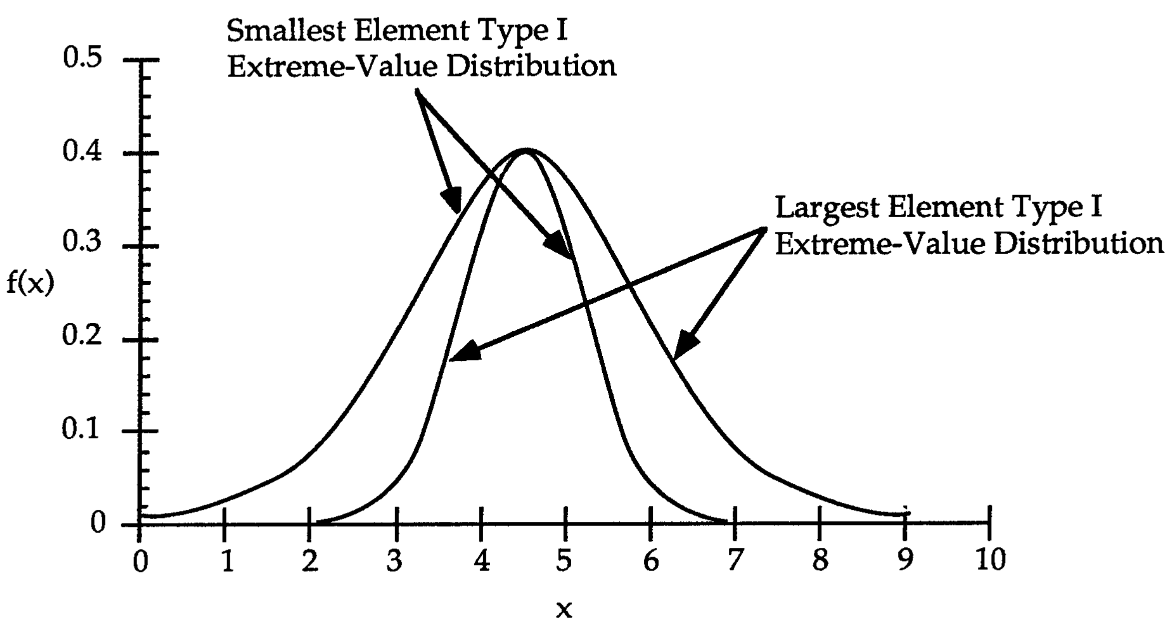


FIGURE 2.9-5: TYPE I EXTREME-VALUE DISTRIBUTIONS FOR SMALLEST AND LARGEST ELEMENTS WITH $\mu = 5$ AND $\sigma = 1$

Notice from viewing Figure 2.9-5 that the Type I Extreme-Value probability density functions for the largest and smallest elements are mirror images of one another. The distribution of maximum values is right skewed, and the distribution of minimum values is left skewed. The hazard function for the smallest element increases exponentially with time and for the largest element the increase is at a decreasing rate approaching a constant value asymptotically.

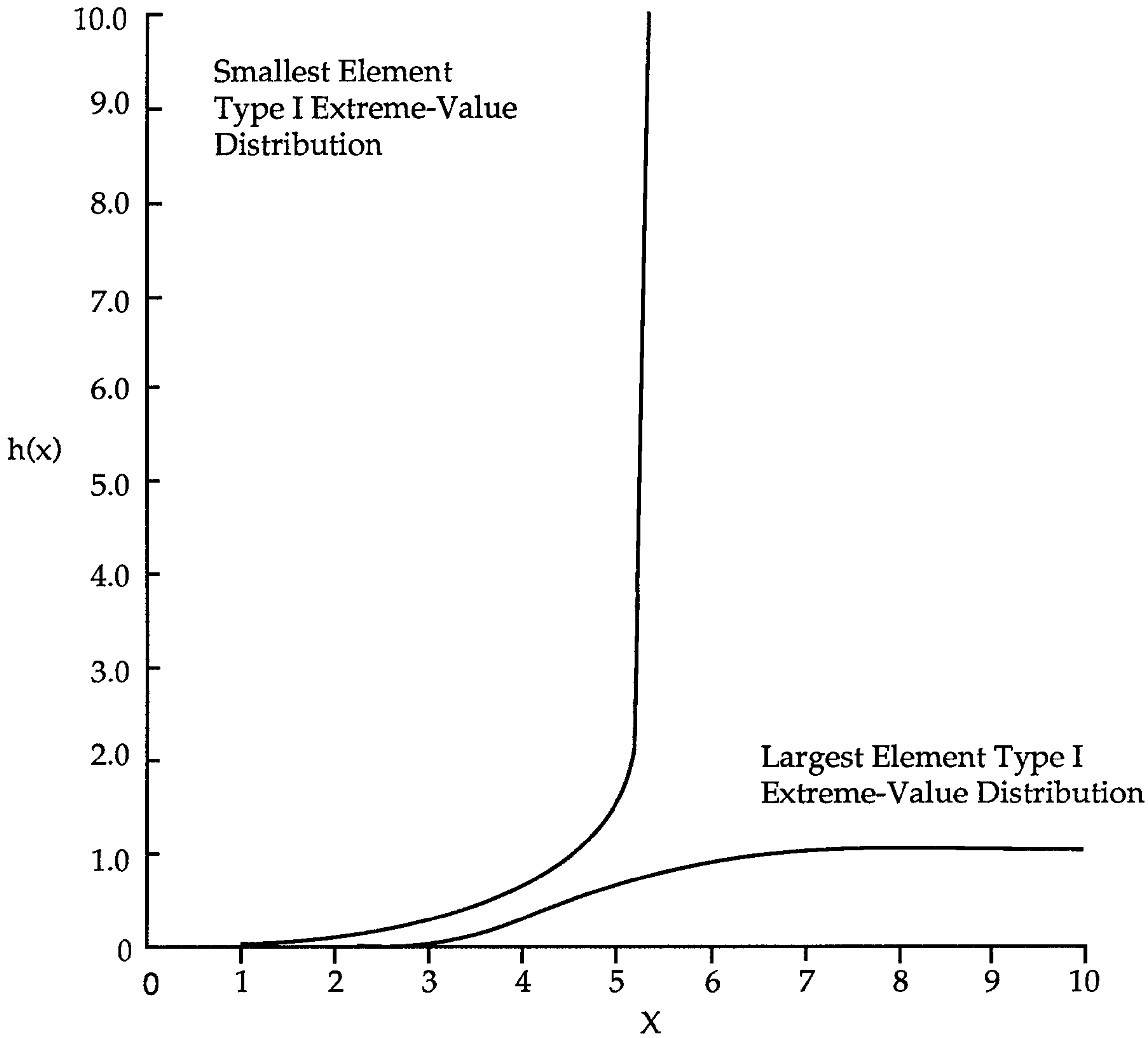


FIGURE 2.9-6: HAZARD FUNCTIONS FOR TYPE I EXTREME-VALUE DISTRIBUTIONS FOR SMALLEST AND LARGEST ELEMENTS WITH $\mu = 5$ AND $\sigma = 1$

2.9.6 Distribution Summary

This section summarizes the major characteristics of each of the five continuous statistical distributions discussed in Section 2.9. Figure 2.9-7 represents the family of statistical distributions and the relationships which exist among them. Table 2.9-2 summarizes the important mathematical functions which are associated with each distribution.

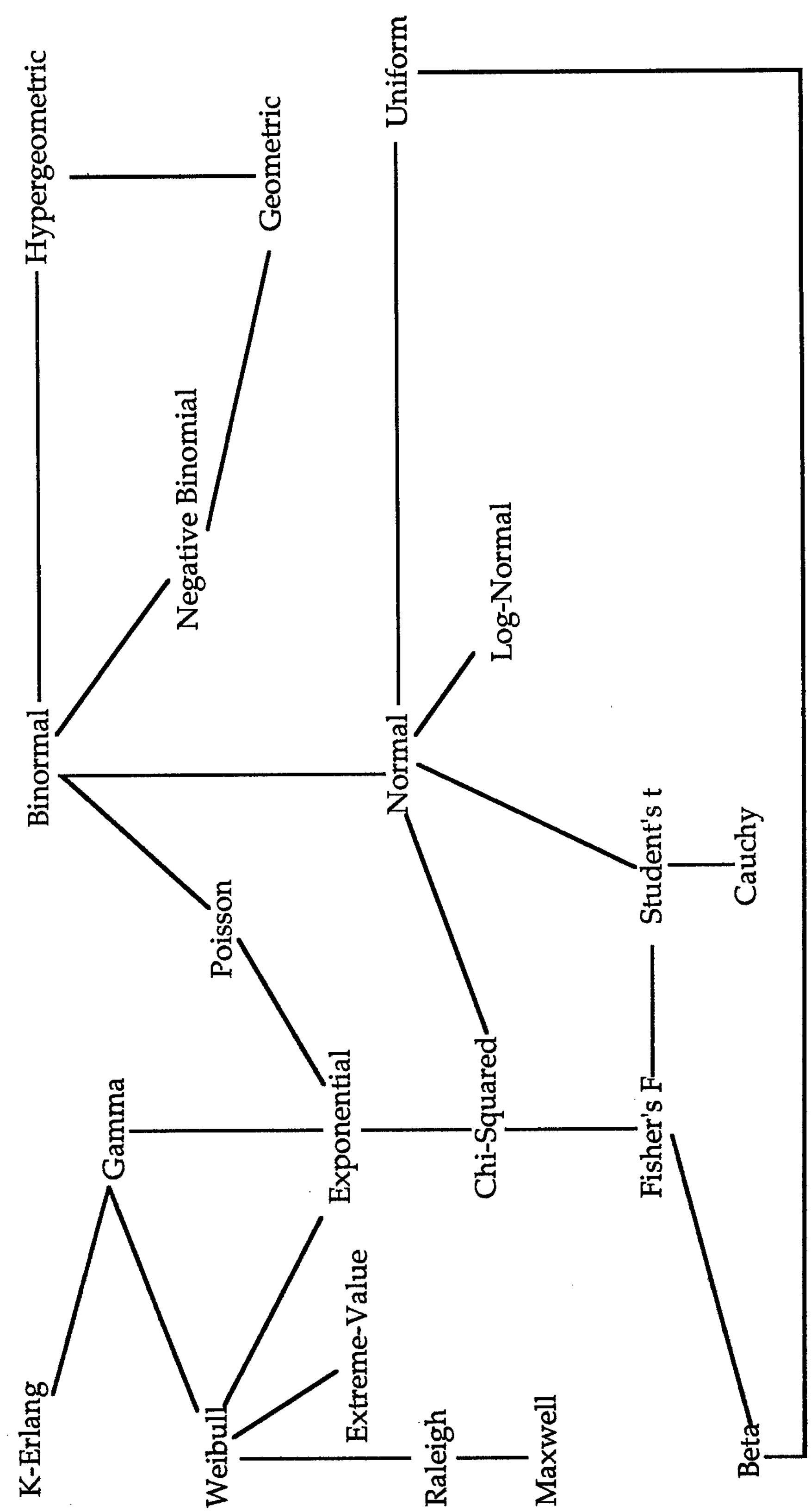


FIGURE 2.9-7: A SUBSET OF THE FAMILY OF DISTRIBUTIONS [5]

TABLE 2.9-2: A SUMMARY OF IMPORTANT DISTRIBUTIONS USED IN RELIABILITY

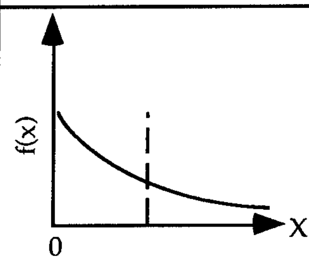
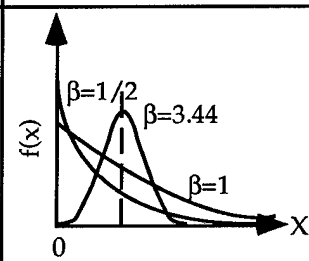
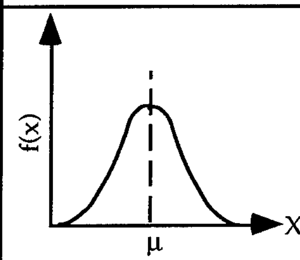
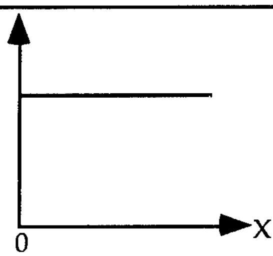
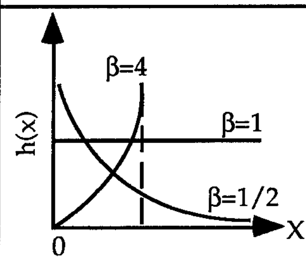
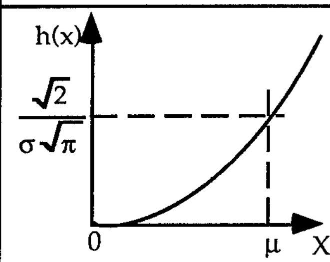
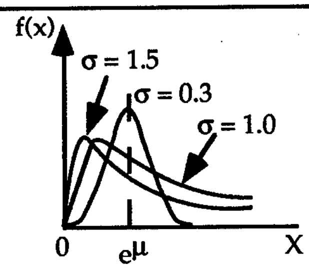
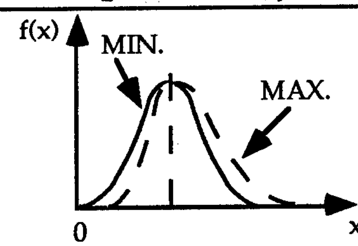
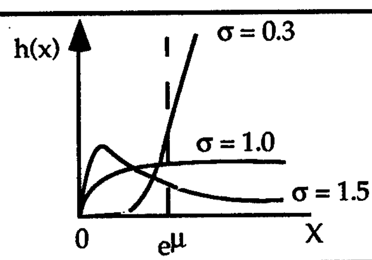
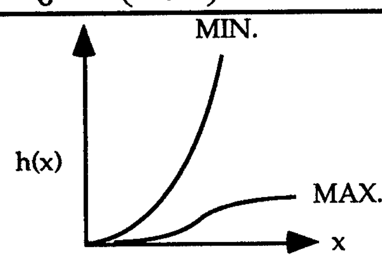
DISTRIBUTION TYPE	EXPONENTIAL	WEIBULL	NORMAL
Range of variate x	$x \geq 0$	$x \geq 0$	$-\infty < x < \infty$
Probability Density Function, f(x)	$f(x) = \lambda \exp(-\lambda x)$	$f(x) = \frac{\beta}{\alpha^{\beta}} \left(x - x_o\right)^{\beta-1} \exp\left[-\left(\frac{x - x_o}{\alpha}\right)^{\beta}\right]$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$
Probability Density Function Graph			
Hazard Function, h(x)	$h(x) = \lambda$	$h(x) = \frac{\beta}{\alpha}\left(\frac{x - x_o}{\alpha}\right)^{\beta-1}$	$h(x) = \dfrac{\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{1 - \phi\left(\frac{x - \mu}{\sigma}\right)}$
Hazard Function Graph			

TABLE 2.9-2: A SUMMARY OF IMPORTANT DISTRIBUTIONS USED IN RELIABILITY (CONT'D)

DISTRIBUTION TYPE	LOG-NORMAL	EXTREME-VALUE
Range of variate x	$0 \leq x \leq \infty$	$-\infty < x < \infty$
Probability Density Function, f(x)	$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right]$	Max. Value: $f(x) = \frac{1}{\sigma} \exp\left\{-\frac{1}{\sigma}(x - \mu) - \exp\left[-\left(\frac{1}{\sigma}\right)(x - \mu)\right]\right\}$ Min. Value: $f(x) = \frac{1}{\sigma} \exp\left\{\frac{1}{\sigma}(x - \mu) - \exp\left[\left(\frac{1}{\sigma}\right)(x - \mu)\right]\right\}$
Probability Density Function Graph		
Hazard Function h(x)	$h(x) = \frac{\frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}}{\sigma x\left(1 - \phi\left(\frac{\ln x - \mu}{\sigma}\right)\right)}$	Max. Value: $h(x) = -\frac{\exp[-(1/\sigma)(x - \mu)]}{\sigma\{\exp[\exp - (1/\sigma)(x - \mu)] - 1\}}$ Min. Value: $h(x) = \frac{1}{\sigma} \exp\left(\frac{x - \mu}{\sigma}\right)$
Hazard Function Graph		

SECTION C

PART RELIABILITY ENGINEERING

3.0 EVALUATING THE RELIABILITY OF PARTS

3.1 Mechanisms of Mechanical Failure

The study of mechanical reliability is based on understanding the process of mechanical failure. The process of mechanical failure is described by the failure mechanism. In order for a reliability engineer to be proficient at identifying and describing mechanical failure mechanisms, he may have to draw on experience obtained from many different disciplines such as: fracture mechanics, tribology, material science, physics, chemistry or metallurgy to name just a few. Consider the following: a rolling element bearing race experiences adhesive wear and surface pitting fatigue as a result of loss of lubrication which results in excessive vibration and noise. In this example, adhesive wear and surface pitting fatigue are the failure mechanisms which describe the process of failure. Loss of lubrication can be identified as the cause of failure. Excessive vibration and noise can be identified as the mode of failure where the mode of failure describes the effect of failure on the function of a part. Example failure mode distributions are illustrated in Table 3.1-1 for eleven different device types. A full complement of generic failure mode distributions are presented in Reference [24]. So, each mechanical failure can be characterized by: a cause, a mechanism (process) and a mode (relative consequence) of failure.

Most mechanisms of mechanical failure can be categorized by one of the following mechanical failure processes:

Failure by:
a) Distortion
b) Fracture
c) Wear
d) Corrosion

These "macro-mechanisms" represent the broadest class of failure mechanisms. Each macro-mechanism contains a number of more specific failure mechanisms, some of which are presented in Table 3.1-2.

TABLE 3.1-1: NORMALIZED FAILURE MODE DISTRIBUTIONS

Device Type	Failure Mode	Failure Mode Probability (α)
Accumulator	Leaking	.47
	Seized	.23
	Worn	.20
	Contaminated	.10
Actuator	Spurious Position Change	.36
	Binding	.27
	Leaking	.22
	Seized	.15
Adapter	Physical Damage	.33
	Out of Adjustment	.33
	Leaking	.33
Alarm	False Indication	.48
	Failure to Operate on Demand	.29
	Spurious Operation	.18
	Degraded Alarm	.05
Antenna	No Transmission	.54
	Signal Leakage	.21
	Spurious Transmission	.25
Battery, Lithium	Degraded Output	.78
	Startup Delay	.14
	Short	.06
	Open	.02
Battery, Lead Acid	Degraded Output	.70
	Short	.20
	Intermittent Output	.10
Battery, Rechargeable, Ni-Cd	Degraded Output	.72
	No Output	.28
Bearing	Binding/Sticking	.50
	Excessive Play	.43
	Contaminated	.07
Belt	Excessive Wear	.75
	Broken	.25
Blower Assembly	Bearing Failure	.45
	Sensor Failure	.16
	Blade Erosion	.15
	Out of Balance	.10
	Short Circuit	.07
	Switch Failure	.07

TABLE 3.1-2: PRIMARY MECHANISMS OF MECHANICAL FAILURE

Distortion Failure	Fatigue & Fracture	Wear	Corrosion
Buckling	Ductile Fracture	Abrasive Wear (Erosive, Grinding, Gouging)	Corrosion-Fatigue
Yielding	Brittle Fracture	Adhesive Wear (Galling)	Stress-Corrosion
Creep	Fatigue Fracture	Subsurface - Origin Fatigue	Galvanic Corrosion
Creep Buckling	High-Cycle Fatigue	Surface-Origin Fatigue (Pitting)	Crevice Corrosion
Warped	Low-Cycle Fatigue	Subcase-Origin Fatigue (Spalling)	Pitting Corrosion
Plastic Deformation (Permanent)	Residual Stress Fracture	Cavitation	Biological Corrosion
Elastic Deformation (Temporary)	Embrittlement-Fracture	Fretting Wear	Chemical Attack
Thermal Relaxation	Thermal Fatigue Fracture	Scoring	Fretting Corrosion
Brinnelling	Torsional Fatigue		
	Fretting Fatigue		

Distortion failures are characterized by either a permanent or temporary change in either the size or shape of a part which prevents the part from performing its intended function. Since engineering materials have various degrees of elasticity, they are expected to distort under load or change in temperature. But when the magnitude of this distortion exceeds certain limits, the integrity and function of the part can be compromised thus causing failure. Examples of distortion failures include: yielding, creep (gradual distortion) and buckling (compression instability). Each of these examples describes the failure mechanism or failure process.

In the case of fatigue and fracture, wear and corrosion, numerous references such as those on fracture mechanics (Reference [96]) and tribology (Reference [95]) discuss these mechanisms in greater detail than is appropriate here and the reader is referred to those sources for more information.

3.2 Mechanical Failure Theories

In this section we will consider two of the more accurate combined stress theories of failure and their importance in design reliability. The two mechanical failure theories to be considered are the:

1) Maximum Normal Stress Theory

2) Distortion Energy Theory

For many design problems, it is found that the level of stress can be utilized by the designer to predict mechanical failure and is of great importance in designing safe, reliable products.

Each of these theories was proposed around the turn of this century to provide models to predict failure at critical points in mechanical parts subjected to a multi-axial state of stress. The multi-axial state of stress at a point is defined in Figure 3.2-1 where stress is the term used to define the magnitude and direction of the internal forces per unit area acting at a given location on a specific plane. Figure 3.2-1 represents an infinitesimal volume ($dx \cdot dy \cdot dz$). The stress represented by the symbol σ defines normal stress which is perpendicular to a cube face. The stress represented by the symbol τ defines the shear stress which is parallel to the face of the cube. In general, the triaxial state of stress at a point can be defined by the nine components of stress as indicated in Figure 3.2-1. The nine components of stress are: τ_{xy}, τ_{yx}, τ_{xz}, τ_{zx}, τ_{yz}, τ_{zy}, σ_x, σ_y and σ_z.

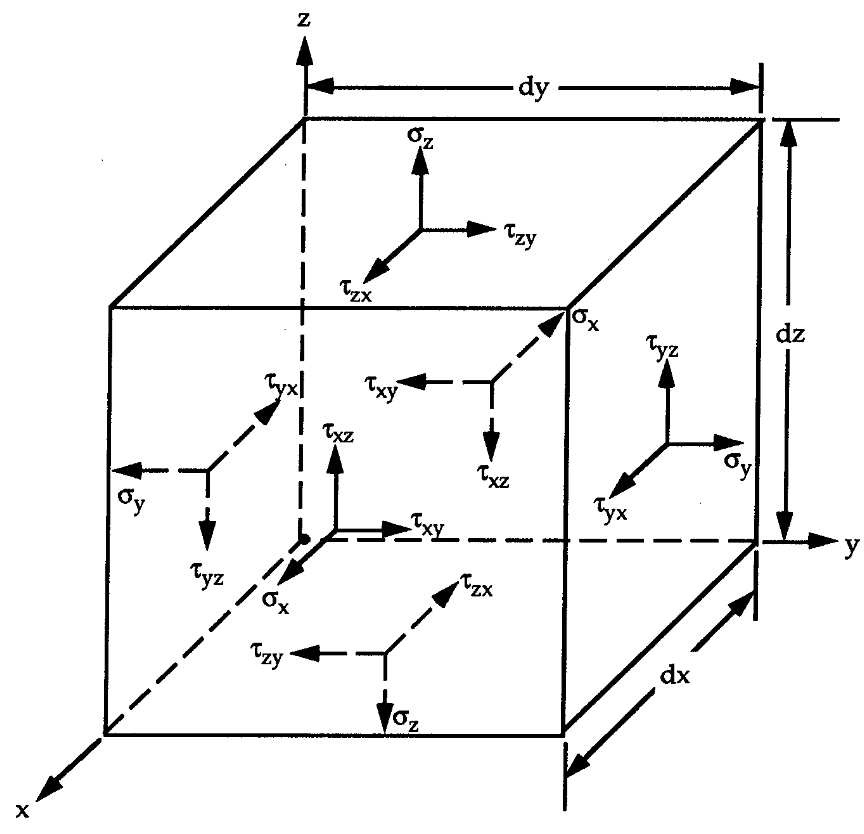


FIGURE 3.2-1: COMPLETE DEFINITION OF THE STATE OF STRESS AT A POINT

For each triaxial stress state there exists an orientation of the infinitesimal volume (dx, dy, dz) where all the shear stress are zero and the normal stresses are a maximum. These maximum normal stresses are called principal stresses and are designated by σ_1, σ_2 and σ_3. These maximum normal or principal stresses will be utilized in our first theory.

3.2.1 The Maximum Normal Stress Theory

The maximum normal stress theory was first proposed by Rankine and is also referred to as Rankine's theory of failure. Rankine formulated his theory by proposing that: "failure is predicted to occur in the multi-axial state of stress when the maximum principal normal stress becomes equal to or exceeds the maximum normal stress at the time of failure in a simple uniaxial stress test using a specimen of the same material." Rankine's theory can be represented mathematically as follows:

Failure is predicted when:

$$\begin{matrix} \sigma_1 \geq \sigma_t & & \sigma_1 \leq \sigma_c \\ \sigma_2 \geq \sigma_t & \text{or} & \sigma_2 \leq \sigma_c \\ \sigma_3 \geq \sigma_t & & \sigma_3 \leq \sigma_c \end{matrix} \tag{3-1}$$

(Note: Recall that compressive stress is negative and tensile stress is positive)

where,

$\sigma_1, \sigma_2, \sigma_3$ = maximum normal stresses
σ_t = tensile yield strength for ductile materials or tensile fracture strength for brittle materials
σ_c = compressive yield strength for ductile materials or compressive fracture strength for brittle material

The utility of the maximum normal stress theory is optimum in predicting the failure of primarily brittle materials such as cast iron. For materials that behave in a ductile fashion, a better choice of failure theories would be the distortion energy theory.

3.2.2 Distortion Energy Theory

The distortion energy theory was first proposed as a failure theory in 1904 by Huber and was later improved by Hencky and VonMises. The distortion energy theory of predicting failure was based on the postulation that the total strain energy per unit volume, u, was composed of two parts; the energy of distortion, u_d, which was the energy associated solely with the change in shape, and the energy of volume, u_v, which was the energy associated solely with the change in volume. Thus, the following relationship was proposed:

$$u = u_v + u_d$$

With these concepts in mind, the distortion energy theory can be stated as follows:

> "Failure is predicted to occur in the multi-axial state of stress when the distortion energy per unit volume, u_d, becomes equal to or exceeds the distortion energy per unit volume at the time of failure in a simple uniaxial stress test using a specimen of the same material."

The final mathematical expression for the distortion energy theory involves the derivation of the energy of volume, u_v, and the total strain energy per unit volume, u, from which the distortion energy, u_d, was derived. A detailed derivation of the distortion energy is presented in Reference [93]. The final mathematical statement of the distortion energy theory of failure is provided in Equation (3-2). Failure is predicted to occur when:

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \geq \sigma_f^2 \tag{3-2}$$

where,

$\sigma_1, \sigma_2, \sigma_3$ = maximum normal stresses
σ_f = uniaxial yield stress

The application of the distortion energy theory for predicting failure has proven most successful when applied to ductile materials.

3.3 Guide to the Application of Part Reliability Prediction Techniques

The discipline of mechanical reliability and reliability prediction of mechanical parts in particular is less standardized than corresponding techniques for electronics. Whether this is good or bad is debatable. The lack of well established techniques has led to innovative approaches which address process and design variability, irregular loading patterns and the effects of maintenance. These approaches to mechanical reliability prediction can serve as better tools to evaluate design integrity. The intent of this section is to provide guidance to practicing reliability engineers and to initiate further dialog concerning the merits of available prediction techniques and approaches associated with mechanical reliability.

Documents such as RADC-TR-83-85, "Reliability Programs for Nonelectronic Designs" and RADC-TR-85-194, "RADC Nonelectronic Reliability Notebook," are good sources of information for mechanical reliability information and guidance. However, little documented guidance exists which specifically discusses procedures for mechanical part reliability prediction. This problem needs to be addressed because quantitative reliability prediction is often imposed as a contractual requirement and the process of reliability prediction, if handled properly, yields useful information for design tradeoff decisions.

To provide some general guidance to practicing reliability engineers, a prioritized list of reliability prediction techniques for mechanical components is presented in Figure 3.3-1. At the top of the pyramid is the analysis of test or historical failure data as the most desirable approach, and at the bottom is the use of surrogate data sources. Other approaches include empirical reliability models and stress/strength interference analysis.

These prediction techniques are part of the mechanical part prediction process. Figure 3.3-2 outlines the flow of a typical mechanical part reliability prediction process. A critical first step in this process is to identify, locate and obtain the major information required for the reliability assessment. Information that is not available to the engineer/analyst will result in assumptions during the assessment which may or may not be valid. The amount of information available will largely depend on the stage of system development (conception, design, production or service). The engineering documentation identified in Figure 3.3-2 provides the analyst with specific information as detailed in Figure 3.3-3.

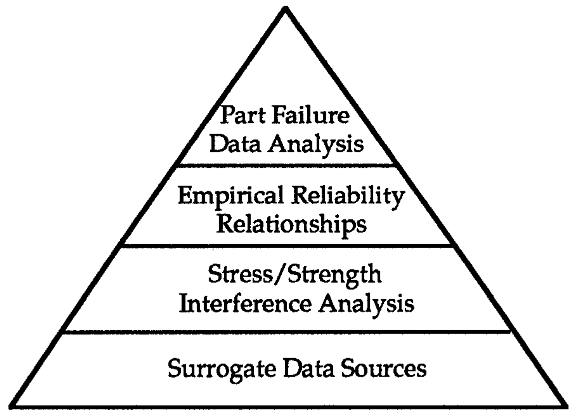


FIGURE 3.3-1: HIERARCHY OF RELIABILITY PREDICTION TECHNIQUES FOR MECHANICAL PARTS

The next stage in the procedure is to select and apply the appropriate part reliability prediction technique(s). Some of the more popular prediction techniques are summarized below:

<u>Part Failure Data Analysis</u>

Analysis of failure data is the preferred approach. Accurate failure data can exist as part of a historical database for systems or equipments which an organization produces, operates or manages. Alternatively, data may exist as a product of a dedicated testing program designed to understand and/or measure the reliability of a new system or component. When data of this nature is available, the underlying time-to-failure distribution should be determined. Use of the Weibull distribution has proven to be particularly effective to characterize the time-to-failure tendencies for mechanical parts. It is also necessary to analyze the failed parts and resulting data in detail to identify trends and to investigate failure mechanisms. In this manner, the reliability engineer can critically evaluate the design integrity and work together with design engineering to improve the design. Refer to Section 3.4 for more detail regarding the analysis of part failure data.

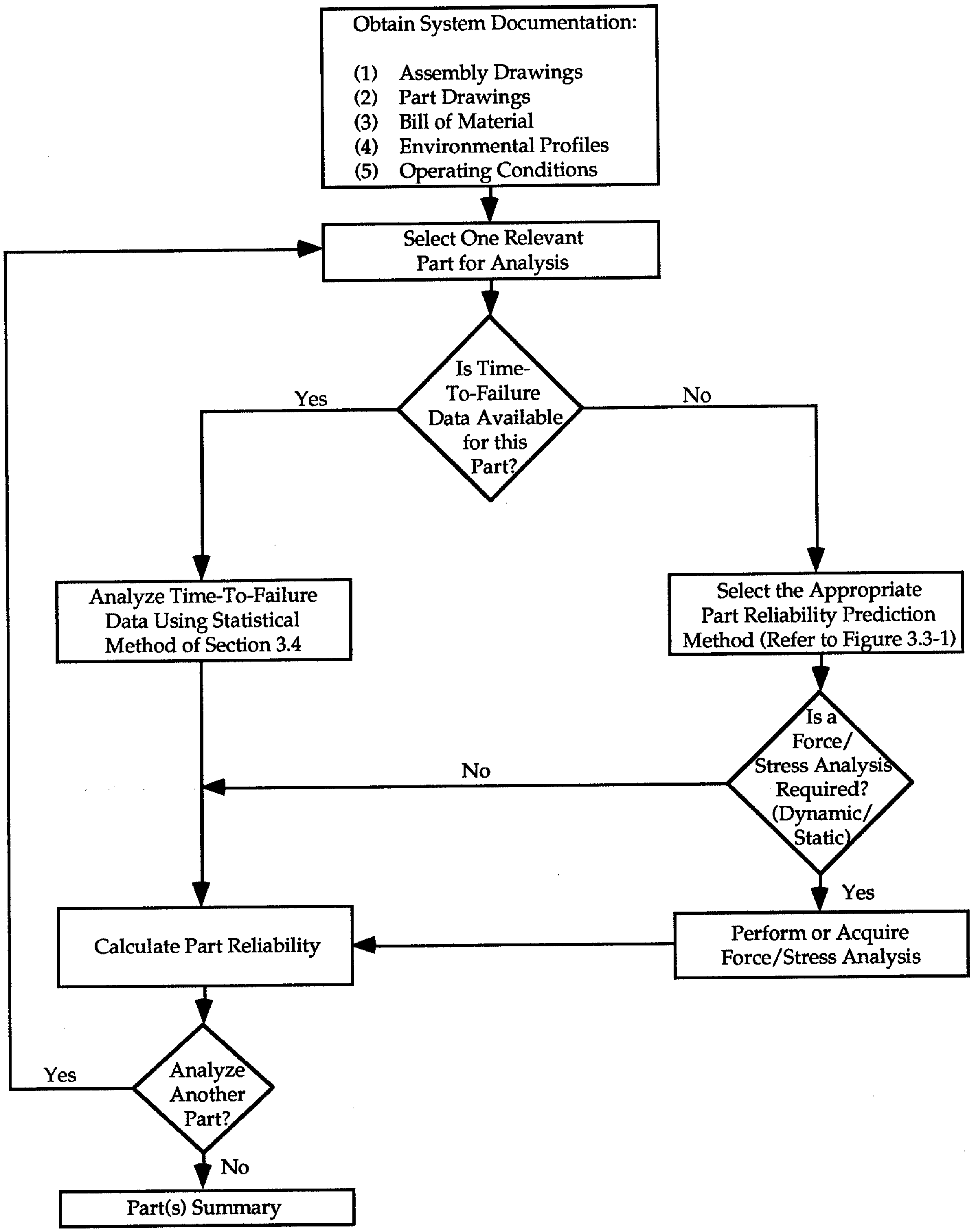


FIGURE 3.3-2: MECHANICAL PART RELIABILITY PREDICTION PROCEDURE

(1) Assembly Drawings

- An Overview of System Operation
- Part Function
- Part Interaction

(2) Part Drawings

- Dimensions
- Material
- Part Description
- Part Numbers

(3) Bill of Material

- A Complete Parts Checklist of the System

(4) Environmental Profiles

- Temperature
- Humidity
- Shock, Vibration
- Atmosphere Contaminants

(5) Operating Conditions

- Loads
- Speeds
- Duty Cycle
- Lubrication

FIGURE 3.3-3: SPECIFIC INFORMATION GAINED FROM DOCUMENTATION

Empirical Reliability Models

Empirical models are generally based on extensive testing for different combinations of loading, materials, dimensions and other physical properties. The models can provide the framework for reliability predictions. Tools required to use these models include the ability to determine some measure of part life (e.g., L_{10} or L_{50} life) and the ability to determine Weibull characteristic life based on part life.

Empirical models often involve computation of an L_{10} life, that is, the time at which 10% of the population will fail. If a Weibull distribution has been determined to be appropriate, then the Weibull scale parameter or characteristic life (α) can be derived using Equation (2-43) and substituting $F(x) = .10$ and $x = L_{10}$ which yields:

$$\alpha = L_{10}\left[\ln\left(\frac{1}{.9}\right)^{-\frac{1}{\beta}}\right] \tag{3-3}$$

where,

α = Weibull characteristic life
L_{10} = time at which 10% have failed (found using the empirical model)
β = Weibull shape parameter

The Weibull hazard rate is then given by,

$$h(x) = \frac{\beta x^{\beta-1}}{\alpha^{\beta}} \tag{3-4}$$

It has been found that for general classes of mechanical components, the Weibull shape parameter remains approximately constant while the characteristic life varies with application stresses, design tolerances, etc. Typical shape parameters (β) have been found to be 2.5 for gears and 1.5 for tapered roller bearings.

The use of empirical reliability models will be discussed in more detail in Section 3.5 of this document.

Stress/Strength Interference Analysis

Stress/strength interference analysis involves the characterization of statistical distributions for the stress acting on a mechanical part and material strength. Historically, stress and strength have been treated as deterministic values in the mechanical design process. The most positive benefit of applying stress/strength interference analysis is the widespread realization that stress and strength are not deterministic values but are subject to variability. By understanding and modeling this variability, the mechanical design process can be improved and made more efficient.

Figure 3.3-4 illustrates the concept of stress/strength interference theory. In approximate terms, the interference or intersection between the two distributions represents the probability of failure. In real terms, the probability of failure is somewhat less than the interference. One problem with stress/strength interference theory is that the intersection between the two distributions can extend far out to the distribution tails. Therefore, if an incorrect underlying distribution was selected or if the variability was not accurately characterized, then the resulting probability of failure may be significantly in error.

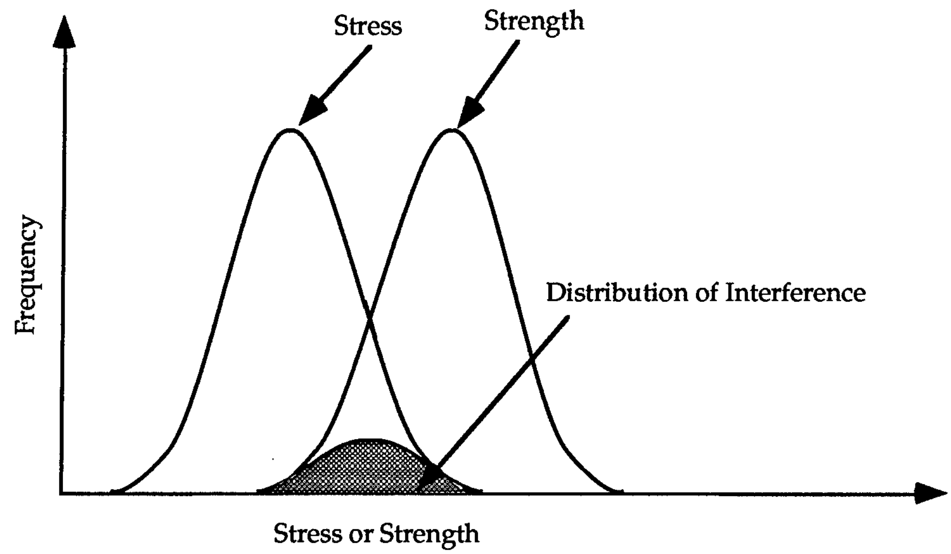


FIGURE 3.3-4: STRESS/STRENGTH INTERFERENCE THEORY

Stress/Strength interference analysis will be discussed in more detail in Section 3.6.

Surrogate Data Sources

Generic failure rate data is available from sources such as RAC Publication NPRD-91, "Nonelectronic Parts Reliability Data" (Reference [30]) and the "RADC Nonelectronic Notebook" (Reference [48]). Data is generally grouped in the form of 'N' failures in 'Y' part hours allowing the computation of an average failure rate. Failure rates from these surrogate data sources are the least desirable method of predicting mechanical component reliability. The average failure rates may not correspond to your particular application and do not account for the possibility of a time dependent hazard rate. Surrogate failure data sources usually assume the exponential distribution to be the representative failure model. This assumption allows a constant hazard rate to be calculated. Surrogate data sources will be discussed in more detail in Section 3.7.

Comparison

Each of the reliability prediction tools described in this Section has merits depending on the particular application, the availability of the required data and the objective of the reliability prediction process. Table 3.3-1 provides a list of the relative advantages and disadvantages of the various techniques. Sections 3.4 through 3.7 will discuss each of these part reliability prediction techniques in detail.

TABLE 3.3-1: COMPARISON OF RELIABILITY PREDICTION TECHNIQUES

Technique	Advantages	Disadvantages
Failure Data Analysis	• Corresponds to actual or simulated loading conditions • Hazard rate time dependency can be analyzed • Comparison and analysis of data can identify design deficiencies and improvements	• Data often not available • Even when available, data is often grouped (i.e., individual time-to-failures not available) • If design is completely new, a dedicated testing program is required which may be expensive
Empirical Reliability Relationships	• Takes advantage of extensive test results • Irregular loading patterns can be accommodated	• Models are available only for a few part types • New processes or materials cannot be accommodated • Models often are for L_{10} life and not hazard rate
Stress/Strength Interference Theory	• Addresses variability of stress and material strength • Quantitative estimates of reliability are available	• Result is presented as a probability of failure instead of a hazard rate • Interference is often at the extremes of the distribution tails • Standard deviation for stress is not always available
Surrogate Data Sources	• Quick and inexpensive • Effective for non-critical or low failure rate components • Easy to combine with electronic predictions	• Constant failure rates assumed • Failure rates are not application sensitive and have limited accuracy • Doubtful that design improvements will result from the prediction process

3.4 Evaluating Part Reliability Using Part Failure Data

3.4.1 Collecting Part Failure Data

Time-to-failure (TTF) data for parts can be obtained in a number of different ways. For example, it can be collected directly from the life testing of many identical parts, or it can be extracted from the failure process of one or many systems which contain the part(s) of interest. However TTF data is collected, the actual operating time of the part should always be specified independent from calendar time. Figure 3.4-1 illustrates the insignificance of a calendar time reference; the calendar time interval may not be proportional to operating TTF.

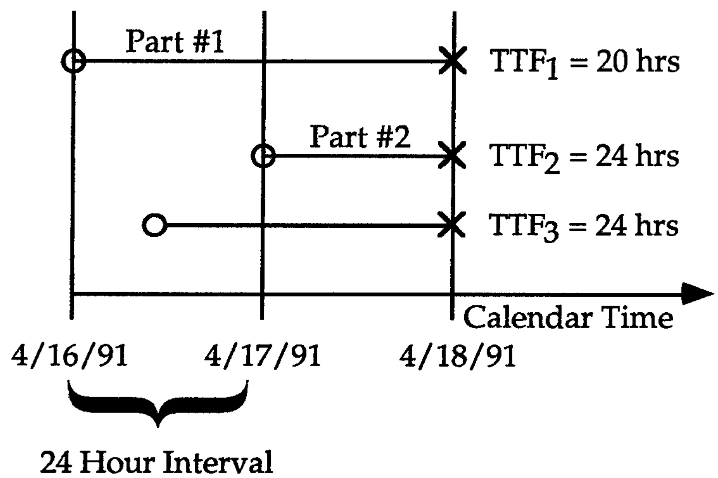


FIGURE 3.4-1: INSIGNIFICANCE OF CALENDAR TIME AS A MEASURE OF PART TIME-TO-FAILURE DATA

All part reliability numerics are generated from the collection and analysis of TTF data. Yet, even collecting this one variable can be an endless task given the infinite number of part types and different applications of all mechanical parts which exist today. Since this information cannot generally be located in a reference source, many engineers are faced with performing life tests and then evaluating the resulting failure data. This section summarizes those procedures required to successfully evaluate resulting failure data.

3.4.2 Order Statistics and Ranking of Part Failure Data

After time-to-failure (TTF) data has been collected on a sample of identical, independent parts, the next step is to determine the appropriate failure model which is representative of the data. This is accomplished utilizing order statistics, ranking, probability plotting and distribution fitting techniques as illustrated in Figure 3.4-2.

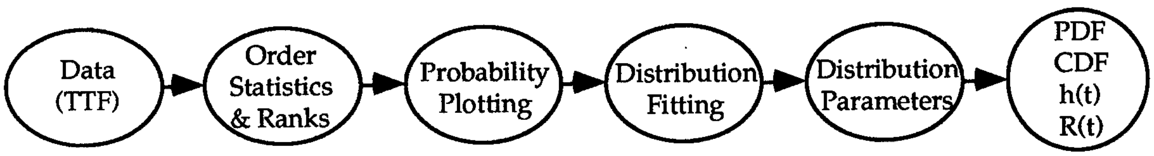


FIGURE 3.4-2: PROCEDURE FOR EVALUATING PART TTF DATA

Order statistics implies that the part times-to-failure are arranged from minimum to maximum and any reference to calendar time is removed and replaced with actual operating time. Since the chronological ordering of part TTF data is not significant, no information is lost and valid statistics can be obtained after a reordering. This same characteristic does not apply to time between failure (TBF) data for a repairable system. Figure 3.4-3 illustrates the application of order statistics to a sample TTF data set.

Original Sample Data Set

Part Number	Life to Failure (10^5 cycles)
1	TTF_1 = 6.6
2	TTF_2 = 1.3
3	TTF_3 = 4.0
4	TTF_4 = 2.7
5	TTF_5 = 5.2
6	TTF_6 = 9.8

Part Number	Life to Failure (10^5 cycles)
2	$TTF_{(1)}$ = 1.3
4	$TTF_{(2)}$ = 2.7
3	$TTF_{(3)}$ = 4.0
5	$TTF_{(4)}$ = 5.2
1	$TTF_{(5)}$ = 6.6
6	$TTF_{(6)}$ = 9.8

FIGURE 3.4-3: APPLICATION OF ORDER STATISTICS FOR PART TTF DATA

The next step in evaluating TTF data is to perform ranking of the order statistics. The purpose of ranking is to determine the cumulative distribution representing the entire population of parts from a limited sample size. In order to do this, the median rank is used. Sample ranking is typically used when the sample size is small (e.g., 1-60). For larger sample sizes, the proportionate cumulative frequency is calculated directly.

Median rank tables can be found in most comprehensive statistics or reliability text books. The median rank can also be calculated directly using Benard's formula:

$$\text{Median Rank} = \frac{j - .3}{n + .4} \tag{3-5}$$

where,

j = failure order number (order statistics applied)
n = sample size

It is important to note that n, the sample size, represents the total number of parts on test, not just the failures. Therefore, if a test is truncated prior to failure of all test specimens, the total number of parts on test is used in Bernard's formula.

The median ranks are calculated for the data set shown in Figure 3.4-3, using Bernard's relationship, and provided in Figure 3.4-4.

Part #	Life to Failure (10^5 cycles)
1	6.6
2	1.3
3	4.0
4	2.7
5	5.2
6	9.8

Apply Order Statistics

Part #	Life to Failure (10^5 cycles)
2	1.3
4	2.7
3	4.0
5	5.2
1	6.6
6	9.8

Apply Median Ranks

Failure Order #	Part #	Life to Failure	Median Rank %
1	2	1.3	10.91
2	4	2.7	26.55
3	3	4.0	42.18
4	5	5.2	57.82
5	1	6.6	73.45
6	6	9.8	89.09

FIGURE 3.4-4: APPLICATION OF MEDIAN RANKS TO ORDER STATISTICS

3.4.3 Preparing Part Failure Data With Suspended Data

Data suspensions often occur when a test is terminated prior to failure of all test items, or in the instance where a failure is inadvertently induced by improper handling or other similar conditions.

In the special case when parts are suspended or have not failed, the failure order number increment (default = 1 for no suspension) can be modified for all failures following the suspended part using the following equation:

$$\text{New Failure Order Increment} = \frac{(n+1)\ -\ \text{previous failure order number}}{1\ +\ \text{number of items following present suspended set}} \tag{3-6}$$

Once the new failure order number is calculated, Benard's formula can be applied to calculate the median rank. This increment is used on all failures following a suspended item until another suspended item is reached.

3.4.4 Weibull Analysis of Part Failure Data

After the median ranks have been established, distribution plotting and fitting techniques are applied. A number of statistical distributions are available as potential failure models. These failure distributions include:

- Weibull
- Normal
- Log-normal
- Exponential

This is by no means a complete list of possible failure distributions, but does include the more popular choices among part data analysts. For our purposes, the Weibull distribution is selected because it represents a family of distributions and can be used to represent or approximate other distributions such as the normal or exponential. The Weibull distribution is currently the most frequently utilized initial failure model for evaluating TTF data from mechanical parts. The probability density function, f(x), for the Weibull distribution is:

$$f(x) = \left[\frac{\beta}{\alpha}\left(\frac{x - x_o}{\alpha}\right)^{\beta-1}\right]\exp\left[-\left(\frac{x - x_o}{\alpha}\right)^{\beta}\right] \tag{3-7}$$

(Note: Other mathematically equivalent forms are available.)

where,

β = Weibull slope or shape parameter
α = Characteristic value or scale parameter ($F(\alpha) = 63.2\%$)
x_0 = Location parameter (expected minimum value)

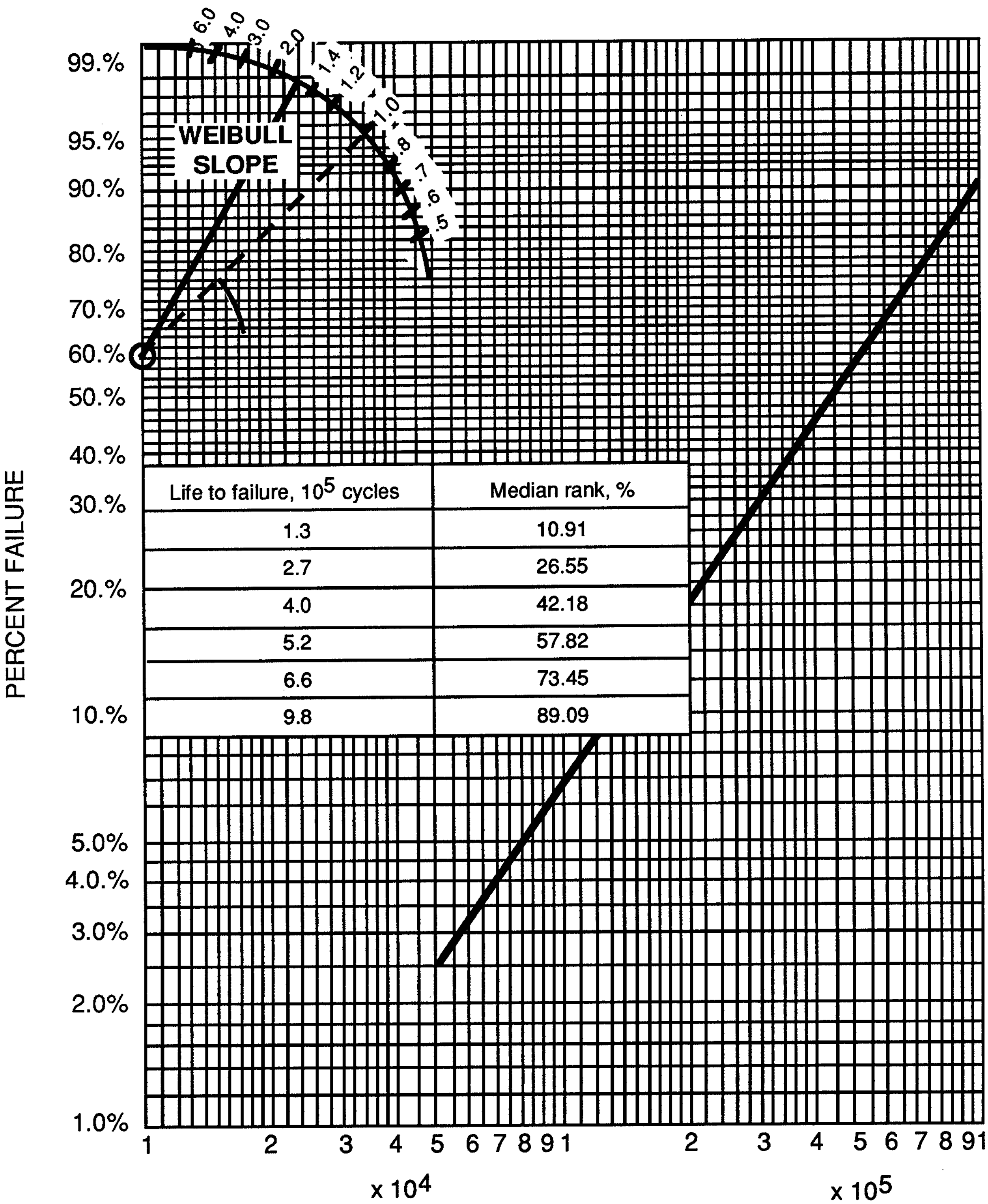
Special probability paper has been developed and is commercially available (Team, Chartwell)[3] which can be used for Weibull probability plotting. Also, commercial statistical packages are available which automate the entire data evaluation process, although utilizing such packages before understanding the theory of the techniques can be hazardous. Figure 3.4-5A shows the time to failure data and associated median ranks to be plotted which were derived in Section 3.4.2 and shown in Figure 3.4-4. These coordinates are plotted on Weibull probability paper and the best fit straight line is drawn through the data points as shown in Figure 3.4-5A. If the data fits a straight line on Weibull probability paper, then the part TTF data is Weibull distributed.

If a straight line can not be reasonably fit through the data points, then the part TTF data is not Weibull distributed or several distinct failure mechanisms are mixed together or the location parameter (x_0) is not zero. Once the line is constructed on Weibull paper. The Weibull slope, β, can be determined from the scales provided on the paper. The methods for doing this vary depending on what paper is used. Most construct either a parallel line as shown in Figure 3.4-5A or a perpendicular line as shown in Figure 3.4-5B to the fitted line through a specified origin point. Each method will be self explanatory by viewing the paper.

The characteristic value, α, is determined from the fitted line. The characteristic value is found by constructing a horizontal line through the ordinate (percent failure) at 63.2%. Then from the point where the horizontal intersects the fitted line, drop a vertical line to the time-to-failure (TTF) axis. The resulting value of time-to-failure is the characteristic value, α. In our case, the data is Weibull distributed with the following parameters:

β = Weibull slope $\cong 1.5$
α = Characteristic value $\cong 5.8 \times 10^5$ cycles (3-8)

3 - Team Graph Papers, Box 25, Tamworth, N.H. 03886; phone: 603-323-8843
- Chartwell Technical Papers, H.W. Peel & Co., Jeymer Drive, Greenford, Middlesex, England; phone: 01-578-6861



Life to failure, 10^5 cycles	Median rank, %
1.3	10.91
2.7	26.55
4.0	42.18
5.2	57.82
6.6	73.45
9.8	89.09

FIGURE 3.4-5A: WEIBULL PROBABILITY PLOTTING ON WEIBULL PROBABILITY PAPER

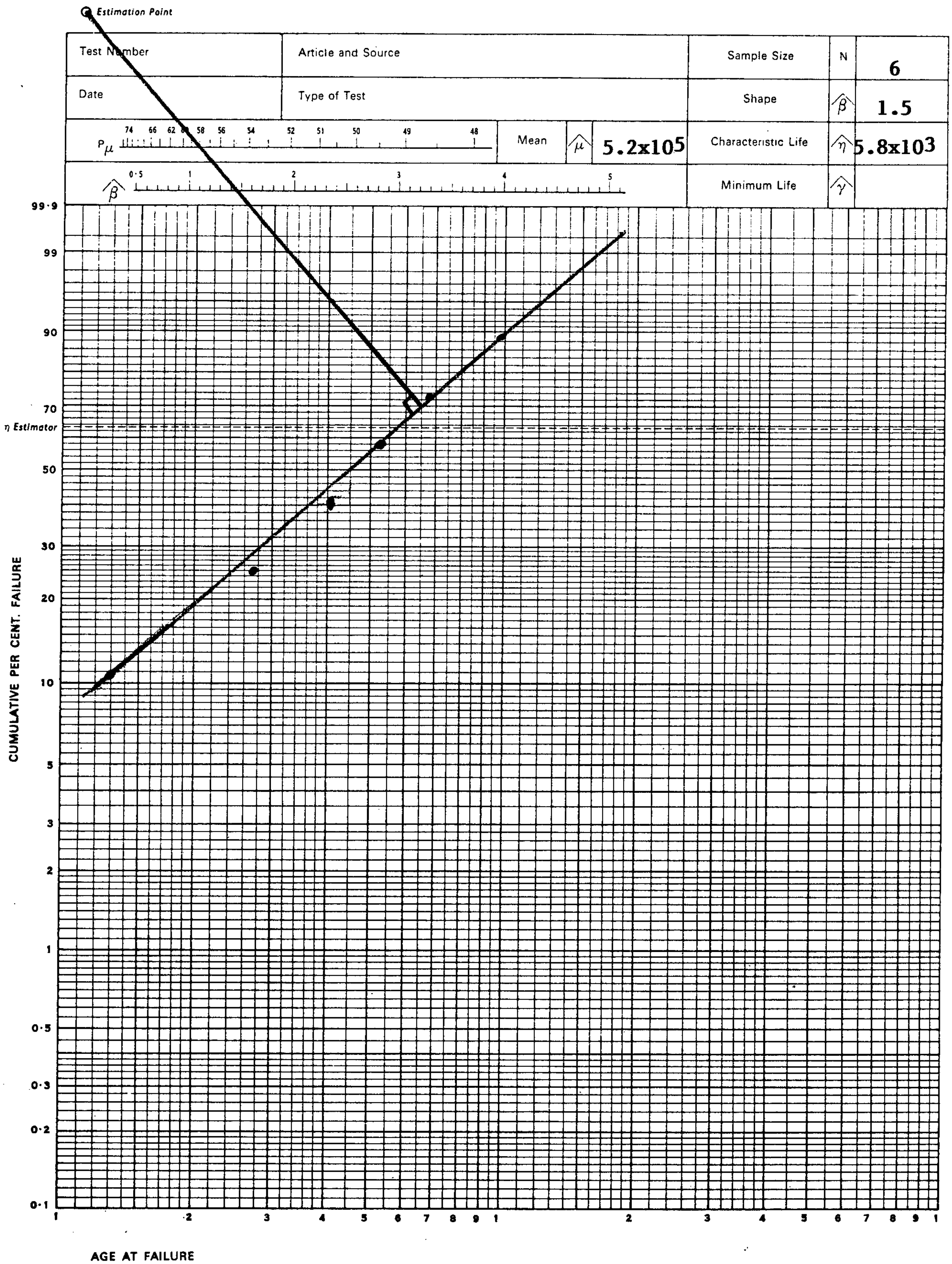


FIGURE 3.4-5B: WEIBULL PLOTTING ON CHARTWELL WEIBULL PAPER

The Weibull mean value (μ_W) for data which is Weibull distributed can be calculated using the following relationship:

$$\mu_W = \alpha \Gamma(1 + (1/\beta)) \tag{3-9}$$

where,

μ_W = Weibull mean value
α = Weibull characteristic value
β = Weibull shape parameter
$\Gamma(x)$ = Gamma function, evaluated at x (Refer to Appendix E)

The mean value is significant because it is the most common measure of central tendency and is useful in characterizing the distribution of failure. The Weibull mean can also be determined directly from the Weibull plot as shown in Figure 3.4-5B but the analyst must realize that the percentile used to estimate mean life is a function of the Weibull shape parameter, β. Table 3.4-1 provides a list of percentiles which may be used to evaluate the Weibull mean. Table 3.4-1 was derived by substituting Equation (3-9) into Equation (2-43) given $x = \mu_W$ then solving for $F(\mu_W)$.

TABLE 3.4-1: RELATION BETWEEN β AND WEIBULL MEAN LIFE

β	**Percentile Used to Estimate Mean Life (see note below)**
0.5	75%
1.0	63.2%
1.5	57.5%
2.0	54.5%
2.5	52.5%
3.0	51%
3.44	50%

Note: $F(\mu_W) = 1 - \exp\left\{-\left[\Gamma(1 + (1/\beta))\right]^{\beta}\right\}$

If we compare the percentile indicated in Figure 3.4-5B which is based on a Weibull shape parameter of 1.5 to the percentile indicated in Table 3.4-1 for $\beta = 1.5$, it is concluded that they are approximately equal (57.5%).

Utilizing Equation (3-9), the mean life can be calculated for the data given in Figure 3.4-5A and Figure 3.4-5B as follows:

$$\mu_W = \alpha\Gamma\left(1 + (1/\beta)\right)$$
$$\mu_W = \left(5.8 \times 10^5\right)\Gamma\left(1 + (1/1.5)\right)$$
$$\mu_W = \left(5.8 \times 10^5\right)\Gamma(1.67)$$
$$\mu_W = 5.24 \times 10^5 \text{ cycles}$$

Notice the value calculated above corresponds to life at 57.5% probability of failure in Figure 3.4-5B.

The final step is to substitute the Weibull parameters derived from Figure 3.4-5A or 3.4-5B back into the probability density function, f(x); the reliability function, R(x); and the hazard function, h(x). Substitution of $\beta = 1.5$ and $\alpha = 5.8 \times 10^5$ into Equation (2-43) yields:

$$f(x) = \frac{1.5\, x^{.5}}{\left(5.8 \times 10^5\right)^{1.5}} \exp\left[-\left(\frac{x}{5.8 \times 10^5}\right)^{1.5}\right] \tag{3-10}$$

Substitution of $\beta = 1.5$ and $\alpha = 5.8 \times 10^5$ into Equation (2-45) yields:

$$R(x) = \exp\left[-\left(\frac{x}{5.8 \times 10^5}\right)^{1.5}\right] \tag{3-11}$$

Substitution of $\beta = 1.5$ and $\alpha = 5.8 \times 10^5$ into Equation (2-46) yields:

$$h(x) = \frac{1.5}{5.8 \times 10^5}\left(\frac{x}{5.8 \times 10^5}\right)^{.5} \tag{3-12}$$

With these functions, we have completed the process shown in Figure 3.4-2. The determination of these functions is sufficient to define the reliability characteristics of the original data set presented in Figure 3.4-3.

3.4.5 Weibull Paper Defined

Since the use of Weibull probability paper is of major importance in the evaluation of part time-to-failure data, it is derived as follows. Recall that the two parameter Weibull cumulative distribution function F(x) is:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \tag{3-13}$$

Equation (3-13) can be rewritten as:

$$\frac{1}{1 - F(x)} = \exp\left(\frac{x}{\alpha}\right)^{\beta} \tag{3-14}$$

Taking natural logarithm of Equation (3-14):

$$\ln \frac{1}{1 - F(x)} = \left(\frac{x}{\alpha}\right)^{\beta} \tag{3-15}$$

Taking the natural logarithm once again:

$$\ln \ln \frac{1}{1 - F(x)} = \beta (\ln x) - (\beta \ln \alpha) \tag{3-16}$$

Equation (3-16) has the form $Y = mX + b$

where,

$$Y = \ln \ln \frac{1}{1 - F(x)}$$

$$X = \ln x$$

$$m = \beta$$

$$b = -\beta \ln \alpha$$

Equation (3-16), therefore, represents a straight line with a slope of β and an intercept, b, on the Cartesian X, Y coordinates. Hence, Weibull probability paper is a plot of:

$$Y = \ln \ln \frac{1}{1 - F(x)}$$

$$X = \ln x$$

Chartwell Weibull probability paper available from H.W. Peel & Company Ltd. (Middlesex, England) is shown in Figure 3.4-6. An advantage of utilizing Chartwell Weibull paper is that it contains a scale where the cumulative percentage associated with the mean can be read directly.

3.5 Failure Rate Models and Life Assessment Models Used for Evaluating Part Reliability

3.5.1 Base Failure Rate With Adjustment Factors Models

The base failure rate with adjustment factors model follows the generalized form:

$$\lambda_p = \lambda_b \prod_{i=1}^{n} \pi_i \qquad (3\text{-}17)$$

where,

λ_p = predicted part failure rate
λ_b = base failure rate
π_i = adjustment factors
n = number of required adjustment factors

These are not statistical models; rather engineering approximations.

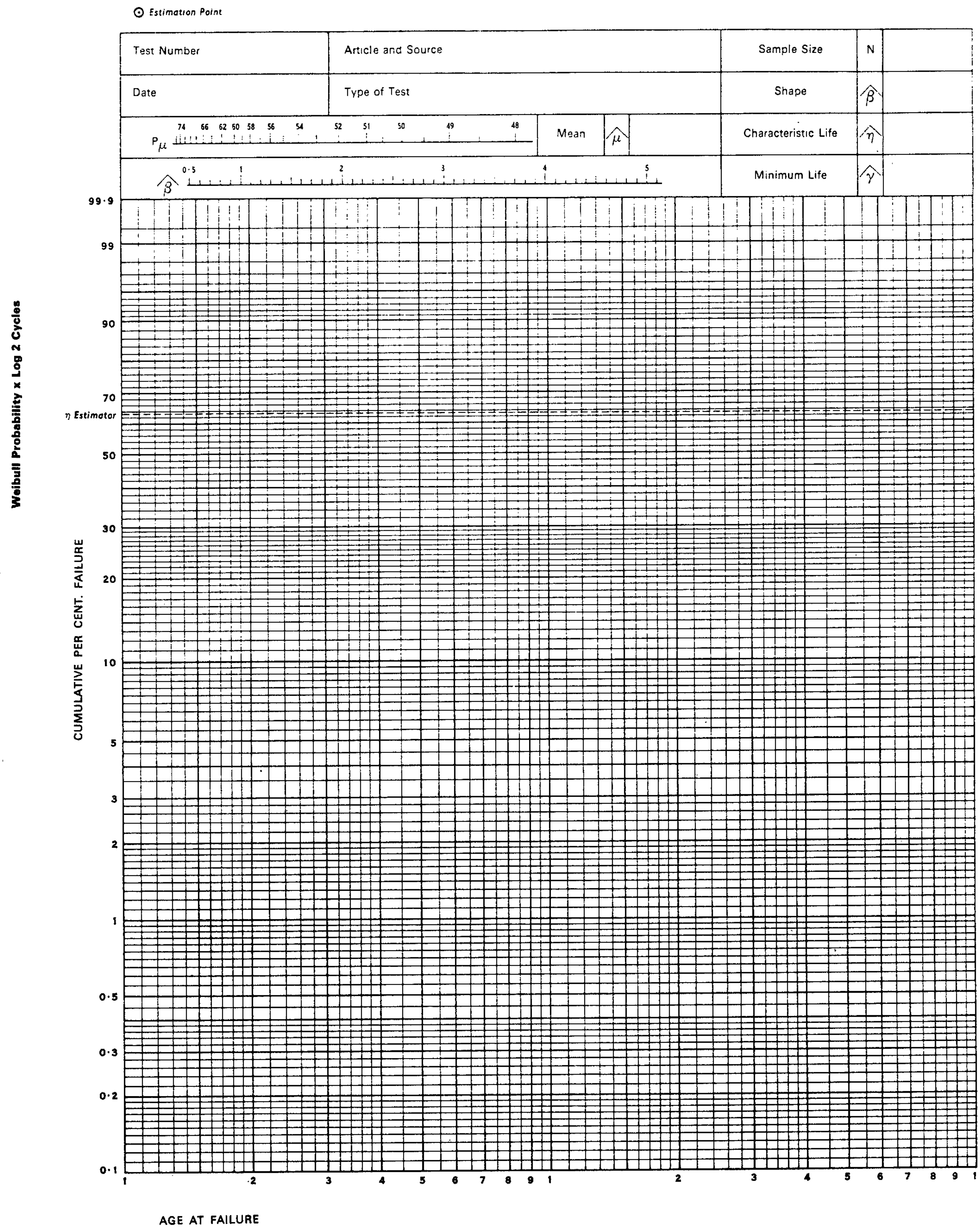


FIGURE 3.4-6: WEIBULL PROBABILITY PAPER

The following list of terms are typically required when utilizing the model of Equation (3-17) to predict the failure rates:

- Part environment
- Part construction
- Part operating conditions
- Part quality level

Currently, only a limited number of models of this type are available for predicting the constant failure rate of mechanical and electromechanical devices. The two main sources containing developed models are References [43] and [46].

The base failure rate with adjustment factors method for predicting part reliability is a well-established method and the calculations are simple to apply. Both of these features are advantages when using this method. The disadvantages of applying this method stem from the fact that only a few models are currently available and they tend to disregard the time dependent nature of mechanical part hazard rates.

MIL-HDBK-217F "Reliability Prediction of Electronic Equipment" (Version F discussed here) is a military handbook that uses the model of Equation (3-17). MIL-HDBK-217's primary concern is electronic devices but some electromechanical devices are also considered, such as:

- Synchros and resolvers
- Elapsed time meters
- Relays
- Switches
- Connectors
- Transformers
- Coils
- Lasers

MIL-HDBK-217 provides the necessary tables to identify the correct base failure rate (λ_b) and adjustment factors (π_i) to calculate the predicted failure rate given specific physical characteristic and operational conditions of the particular device.

MIL-HDBK-217 has also adopted a failure rate prediction model for fractional horsepower motors utilizing rolling element grease packed bearings. This model only accounts for bearing and winding failures. A large body of time-to-failure data was available from which this model was developed. Bearing and winding failures accounted for better than 80% of all failures. Attempts to include other causes in the model failed to produce significant improvements. Application of the model to D.C. brush motors assumes that the brushes are inspected and replaced and are not a failure item. The model was developed at Shaker Research Corporation by D.S. Wilson and R. Smith and is summarized in the technical report: RADC-TR-77-408, entitled Electronic Motor Reliability Model. The model adopted by MIL-HDBK-217 is a simplified version of a more complex model. The simplified failure rate model is given as:

$$\lambda_p = \left(\frac{x^2}{\alpha_B^3} + \frac{1}{\alpha_w}\right) x\ 10^6 \text{ (failures / } 10^6 \text{ hours)} \tag{3-18}$$

where,

λ_p = average failure rate (failures/ 10^6 hours)

x = motor operating time period, selected by the user, for which average failure rate is calculated (hours). Each motor must be replaced when it reaches the end of this operating period to make the calculated λ_p valid.

α_B = bearing Weibull characteristic life value

α_w = winding Weibull characteristic life value

The bearing and winding Weibull characteristic life values are determined from tables contained in MIL-HDBK-217 and are based on the ambient temperature surrounding each part. Notice that Equation (3-18) does not follow the form of Equation (3-17). The form of Equation (3-18) will be discussed further in Section 3.5.2.

The following example shows the procedural calculation of the predicted failure rate of a toggle switch by the method of MIL-HDBK-217 and Equation (3-17). The tables which are used come directly from MIL-HDBK-217. The example is as follows:

Given: A MIL-SPEC toggle switch is used in a ground fixed environment. The switch is a snap-action and is single-pole, double-throw. It is operated on the average of one cycle per hour, and the load current is 50 percent of rated and is resistive.

Find: The failure rate of the switch.

Step 1: The base failure rate (λ_b) is found in Table 3.5-1 and is determined to be 0.00045 failures/10^6 hours

Step 2: The environmental factor π_E for ground fixed environment is determined from Table 3.5-2 to be 3.0.

TABLE 3.5-1: BASE FAILURE RATES, λ_b, FOR SWITCHES

Description	MIL-SPEC	Lower Quality
Snap-action	0.00045	0.034
Non-snap action	0.0027	0.04

TABLE 3.5-2: ENVIRONMENTAL FACTOR - π_E

Environment	π_E
G_B	1.0
G_F	3.0
G_M	18
N_S	8.0
N_U	29
A_{IC}	10
A_{IF}	18
A_{UC}	13
A_{UF}	22
A_{RW}	46
S_F	.50
M_F	25
M_L	67
C_L	1,200

Step 3: The contact from factor π_C is determined from Table 3.5-3. For a single-pole, double-throw switch, π_C is 1.7.

Step 4: The cycling factor π_{cyc} is determined from Table 3.5-4 to be equal to 1.0.

Step 5: The stress factor π_L from Table 3.5-5 for 50 percent stress factor and a resistive load is determined to be 1.48.

TABLE 3.5-3: π_C FACTOR FOR CONTACT FORM AND QUALITY

Contact Form	π_C
SPST	1.0
DPST	1.5
SPDT	1.7
3PST	2.0
4PST	2.5
DPDT	3.0
3PDT	4.2
4PDT	5.5
6PDT	8.0

TABLE 3.5-4: π_{CYC} FACTOR FOR CYCLING RATES

Switching Cycles per Hour	π_{CYC}
< 1 cycle/hour	1.0
> 1 cycle/hour	number of cycles/hour

TABLE 3.5-5: π_L STRESS FACTOR FOR SWITCH CONTACTS

Stress S	Load Type		
	Resistive	Inductive	Lamp
0.05	1.00	1.05	1.06
0.1	1.02	1.06	1.28
0.2	1.06	1.28	2.72
0.3	1.15	1.76	9.49
0.4	1.28	2.72	54.6
0.5	1.48	4.77	
0.6	1.76	9.49	
0.7	2.15	21.4	
0.8	2.72		
0.9	3.55		
1.0	4.77		

where,

$$S = \frac{\text{operating load current}}{\text{rated resistive load current}}$$

Step 6: The failure rate mathematical model for toggle switches is given by:

$$\lambda_p = \lambda_b (\pi_E \times \pi_C \times \pi_{cyc} \times \pi_L)$$

Substituting the value for these factors yields the failure rate.

$$\lambda_p = 0.00045 (3.0 \times 1.7 \times 1.0 \times 1.48)$$

$$\lambda_p = 0.0034 \text{ failures}/10^6 \text{ hours}$$

3.5.2 Average Cumulative Hazard Rate Analysis

The hazard rates experienced by many mechanical parts are not constant but rather vary as a function of time. The average cumulative hazard rate method reduces a time dependent hazard rate into a single average failure rate which can be treated as a constant failure rate for a specified time interval. This constant failure rate can then be used in constant failure rate reliability predictions. A disadvantage of this method is it reduces the accuracy of the original failure model to that of a constant hazard rate over the specified time interval.

The following is a list of items required when utilizing the average cumulative hazard rate method for components exhibiting a Weibull failure distribution:

(a) Weibull slope or shape parameter, β

(b) Component service life, x (e.g., warranty time, time-to-overhaul, design life)

(c) A point estimate of life that a stated percentile of the component population will complete or exceed without failure (e.g., L_{10} life or Weibull characteristic life)

The conditional hazard rate, h(x) was presented in Section 2.8. The hazard rate was defined in Equation (2-23) as:

$$h(x) = \frac{f(x)}{1 - F(x)}$$

An average failure rate (λ_{avg}) can be determined for the purpose of comparison with other constant failure rate components. The average failure rate over an interval x_1 to x_2 can be defined as:

$$\lambda_{avg} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} h(x)dx \tag{3-19}$$

If the average failure rate to any time (x) from time zero is utilized, Equation (3-19) reduces to:

$$\lambda_{avg} = \frac{1}{x} \int_{0}^{x} h(x)dx \tag{3-20}$$

However, the integral of the hazard function, h(x), is the cumulative hazard function, H(t). Equation (3-20) becomes:

$$\lambda_{avg} = H(x) / x = \overline{H}(x) \tag{3-21}$$

Equation (3-21) represents the average cumulative hazard rate or the average failure rate based on the condition that the part has not yet failed to time x.

The hazard function, h(x), corresponding to the Weibull distribution was presented in Section 2.9.2, Equation (2-46) as:

$$h(x) = \frac{\beta}{\alpha} \left(\frac{x - x_o}{\alpha} \right)^{\beta - 1} \tag{3-22}$$

Substituting Equation (3-22) into Equation (3-20) and letting $x_o = 0$ yields the average cumulative hazard function or the average failure rate for the Weibull distribution:

$$\lambda_{avg} = \overline{H}(x) = x^{\beta-1} / \alpha^{\beta} \tag{3-23}$$

The utility of Equation (3-23) was mentioned earlier in Section 3.5.1, Equation (3-18). Notice that the motor model, Equation (3-18) of MIL-HDBK-217, has the form of Equation (3-23). Substitution of β equal to 3 into Equation (3-23) yields the bearing portion of the model and β equal to 1 yields the winding portion of the motor model. This motor model is an example of combining average failure rates to develop a competing risk model.

The average cumulative hazard rate can only be used effectively if the limitation of this reliability measure is understood. The primary limitation of this method is:

- The average cumulative hazard rate provides a valid approximation over relatively short intervals of the part service life. The average cumulative hazard rate is only a gross approximation and should only be used when a constant failure rate measure is required.

3.5.3 Life Assessment Model For Rolling Element Bearings

Using the average cumulative hazard rate method to determine the average failure rate of a rolling element bearing is now considered. The following five steps are used to calculate the average failure rate for a double rowed tapered roller bearing subject to a purely radial load, F, of 1,000 pounds and a speed of 60,000 revolutions per hour. The basic dynamic capacity, C, of the bearing is 11,700 pounds. Determine the average failure rate for 50,000 hours of constant operation.

Step 1: Identify the following bearing characteristics:

(a) double rowed tapered roller bearing

(b) basic dynamic capacity (from manufacturer) is 11,700 pounds

(c) Weibull slope (β) for tapered roller bearing is 1.5 (manufacturer)

(d) load life exponent (from empirical data) is 10/3 for roller bearings

Step 2: The standard equation for determining the L_{10} life of bearings, which can be found in most references in bearing selection, was developed in the 1940s by Lundberg and Palmgren and has been the accepted criterion by the rolling element bearing industry since 1947 when their papers were published. The L_{10} life equation is given in Equation (3-24):

$$L_{10} = \left(\frac{C}{F}\right)^{P} \times 10^{6} \text{ (revolutions)} \tag{3-24}$$

where,

L_{10} = the number of revolutions that 90% of a population of bearings will complete or exceed without failure

C = the basic load rating in pounds
F = the equivalent radial load in pounds
P = 3 for ball bearings, 10/3 for roller bearings (from empirical data)

Equation (3-24) can be used to determine the point estimate of bearing life that 90% of the population will complete or exceed without failure:

$$L_{10} = \left(\frac{11{,}700}{1{,}000}\right)^{10/3} \times 10^6$$

$$L_{10} = 3.6 \times 10^9 \text{ (revolutions)}$$

Step 3: Convert L_{10} (revolutions) to L_{10} (hours) given the speed of 60,000 revolutions per hour:

$$L_{10}\text{ (hours)} = L_{10}\text{ (revolutions)} / \text{revolutions per hour}$$

$$L_{10}\text{ (hours)} = 3.6 \times 10^9 / 60{,}000 \tag{3-25}$$

$$L_{10}\text{ (hours)} = 60{,}000$$

Step 4: A graphical method using L_{10} (hours) and the Weibull slope (β) is applied to determine the Weibull characteristic life value (α). The Weibull characteristic life is defined as the life that 36.8% of the population will complete or exceed without failure. Figure 3.5-1 shows the point/slope analysis using Weibull probability paper. The derivation of the characteristic life percentile and the Weibull axis parameters is given in Appendix A. The Weibull characteristic life value is approximated to be 2.7×10^5 hours.

The Weibull characteristic life value can also be derived analytically from the Weibull cumulative density function as:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \tag{3-26}$$

$$\exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] = 1 - F(x)$$

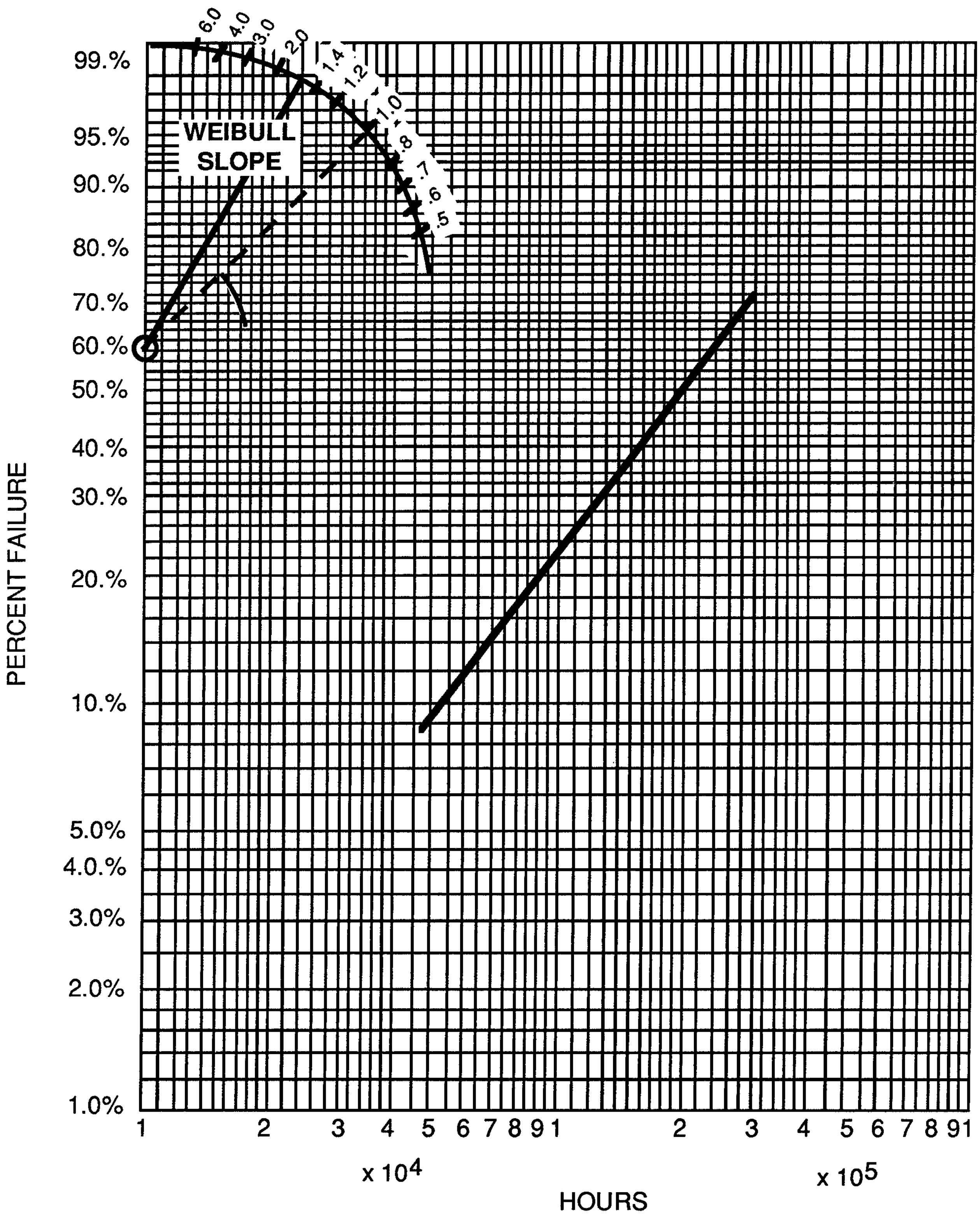


FIGURE 3.5-1: WEIBULL POINT/SLOPE ANALYSIS TO DETERMINE THE WEIBULL CHARACTERISTIC LIFE VALUE (α)

$$\left(\frac{x}{\alpha}\right)^{\beta} = -\ln\,[1 - F(x)]$$

Substitution of the L_{10} life and the Weibull slope (β) yields:

$$\alpha = \frac{60{,}000}{\{-\ln\,[1 - .10]\}^{1/1.5}}$$

$$\alpha = 268{,}967 \text{ (hours)}$$

Step 5: Calculate the average failure rate for 50,000 hours of constant operation using Equation (3-23):

$$\lambda_{avg} = H(x) = x^{\beta-1} / \alpha^{\beta}$$
$$\lambda_{avg} = (50{,}000)^{.5} / (2.7 \times 10^{5})^{1.5}$$
$$\lambda_{avg} = 1.6 \times 10^{-6} \text{ (failures / hour)}$$
$$\lambda_{avg} = 1.6 \text{ (failures / } 10^{6} \text{ hours)}$$

3.6 Mechanical Stress/Strength Interference Analysis

Stress/Strength Interference Analysis is a practical engineering tool used for designing and quantitatively predicting the reliability of mechanical components subjected to mechanical loading. The method was originally presented in the following technical reports:

1) Lipson, C., et. al., Reliability Prediction Mechanical Stress/Strength Interference Models, RADC-TR-66-710, March 1967. (AD/813574).

2) Lipson, C., et. al., Reliability Prediction Mechanical Stress/Strength Interference (Nonferrous), RADC-TR-68-403, December 1968. (AD/856021).

These reports are still available and can be obtained from:

National Technical Information Service (NTIS)
Department of Commerce
5285 Port Royal Road
Springfield, VA 22161-2171
(703) 487-4650

or

Defense Technical Information Center (DTIC)
DTIC-FDAC
Cameron Station, Bldg. 5
Alexandria, VA 22304-6145
(703) 274-7633 DSN: 284-7633

The Mechanical Stress/Strength Interference Analysis is a reliability prediction technique which requires probabilistic knowledge of stress and strength in a part at a particular point in time. Application of this method requires that the strength distribution parameters be known. The technical reports mentioned above have extensive fatigue strength distribution tables for numerous ferrous and nonferrous metals. Included are the effects of heat treatment, surface finish, stress concentrators and temperature. The reliability of an unlimited number of mechanical components can be determined using this method. The precision of this method depends to a large extent on the accuracy to which the stress distribution can be estimated. Ways to determine the stress distribution may include actual stress measurement or simulated stress measurement using finite element analysis.

The following is a list of data items required when utilizing Mechanical Stress/Strength Interference Analysis:

a. engineering knowledge of the stress distribution
b. engineering knowledge of the strength distribution
    1. alloy
    2. design life
    3. loading type (axial, bending, torsion)
    4. surface finish (polished, ground, etc.)
    5. heat treatment (annealed, quenched, etc.)
    6. operating temperature
    7. stress concentration factor(s)

The data items in (1) through (7) above can be used to establish the strength distribution.

The method treats both stress and strength as random variables subject to natural scatter. The variability in these two factors results in statistical distributions of stress and strength defined by probability density functions. The area of intersection created by these distributions is referred to as the "interference" which is shown graphically in Figure 3.6-1. If failure is defined by:

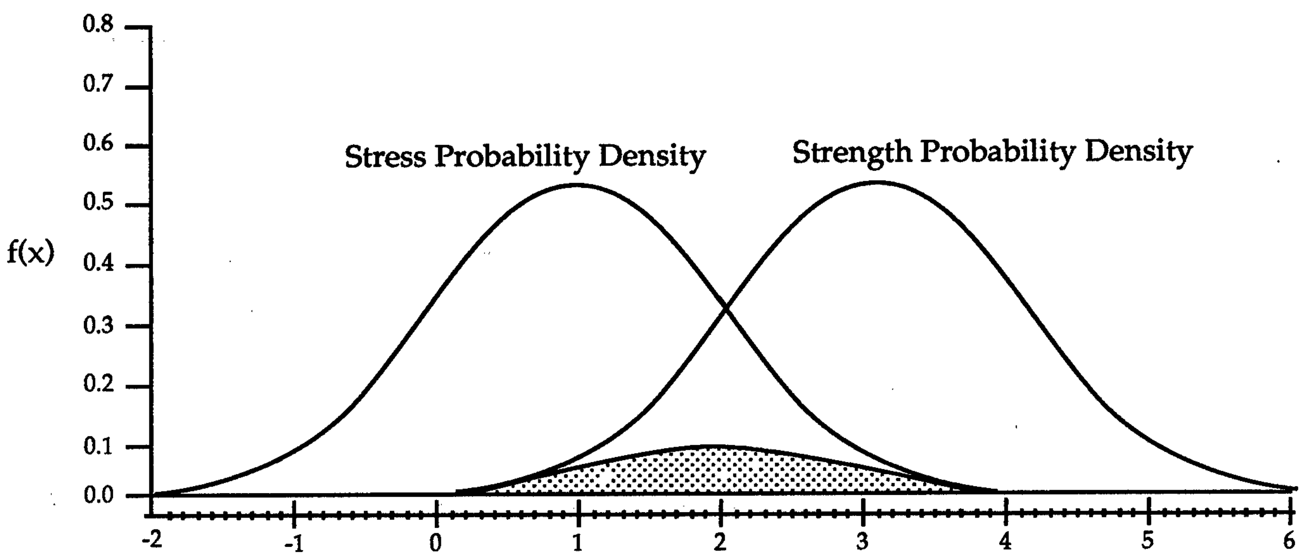
$$(\text{Stress} > \text{Strength}) = \text{failure} \tag{3-27}$$

Interference, when evaluated properly, represents the probability that a random observation from the stress distribution exceeds a random observation from the strength distribution or:

$$P\ (\text{Stress} > \text{Strength}) = \text{interference} \tag{3-28}$$

So, interference represents the probability of failure. The reliability (R) then is expressed as:

$$R = 1 - \text{interference} \tag{3-29}$$



STRESS or STRENGTH
(INTERFERENCE IS THE SHADED AREA)

FIGURE 3.6-1: INTERFERENCE OF STRESS AND STRENGTH DISTRIBUTION [48]

The means of expressing the stress and strength distributions and calculating the resulting interference created by these distributions are contained in RADC-TR-66-710 (Reference [42]) and RADC-TR-68-403 (Reference [41]). In these technical reports, this method is applied to components: a) subjected to completely reversed cyclic bending, axial or torsional loading or b) subjected to a combination of static and cyclic loads.

The original technical reports provide extensive tables giving the interference (probability of failure) as a function of distribution parameters for the following combinations of distribution:

Strength Distribution	Stress Distribution
Weibull	Normal
Weibull	Weibull
Normal	Normal
Largest Extreme-Value	Normal
Smallest Extreme-Value	Normal

The assumption of a normal stress and strength distribution is by far the most popular in discussions of stress/strength inference analysis because the mathematics is easily managed. But an assumption of a normally distributed stress or strength distribution should always be justified with sound engineering rationale.

3.6.1 Application of Stress/Strength Interference

Application of the Stress/Strength Interference Method for estimating the reliability of mechanical components takes several forms depending upon the choice of stress and strength distributions. Numerical examples illustrating the use of this method are presented in this section. Further examples can be found in the technical reports RADC-TR-66-710, RADC-TR-68-403 and also in the document Nonelectronic Reliability Notebook (RADC-TR-85-194). Three separate cases are considered:

Case 1: The assumption is made that the standard deviation of the service stress distribution is negligible compared to the mean stress. The fatigue strength distribution parameters are selected from the alloy tables in the source documents. A typical Case 1 situation is shown in Figure 3.6-2.

Case 2: The assumption is made that the service stress has a normal distribution and the standard deviation is a fixed percentage of the mean. The fatigue strength distributions and their parameters are selected from the alloy tables in the source documents. A typical Case 2 situation is shown in Figure 3.6-3.

Case 3: To use the interference method, a simple completely reversed cyclic stress must be stated. Case 3 considers an example when a cyclic stress has a variable magnitude (stress spectrum). A reduction method is applied which converts the stress spectrum to a simple cyclic equivalent $\left(S_{equ}\right)$ as graphically represented in Figure 3.6-4. The fatigue strength distributions and their parameters are selected from the alloy tables in the source documents.

A numerical example of each of the three cases are now presented:

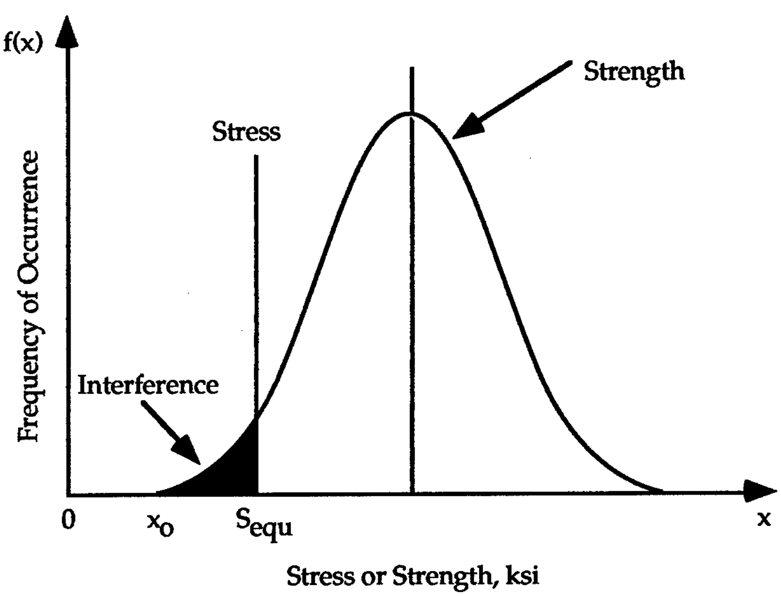


FIGURE 3.6-2: INTERFERENCE WITH STANDARD DEVIATION OF STRESS EQUAL TO ZERO [42]

Case 1: Example

A cylindrical component is to be subjected in service to completely reversed bending stresses of ±23.6 ksi at ambient temperature. The design life is 10^6 cycles. Estimate the reliability if the component is constructed of hot rolled aluminum alloy 2014, heat-treated to the T6 condition, and mechanically polished.

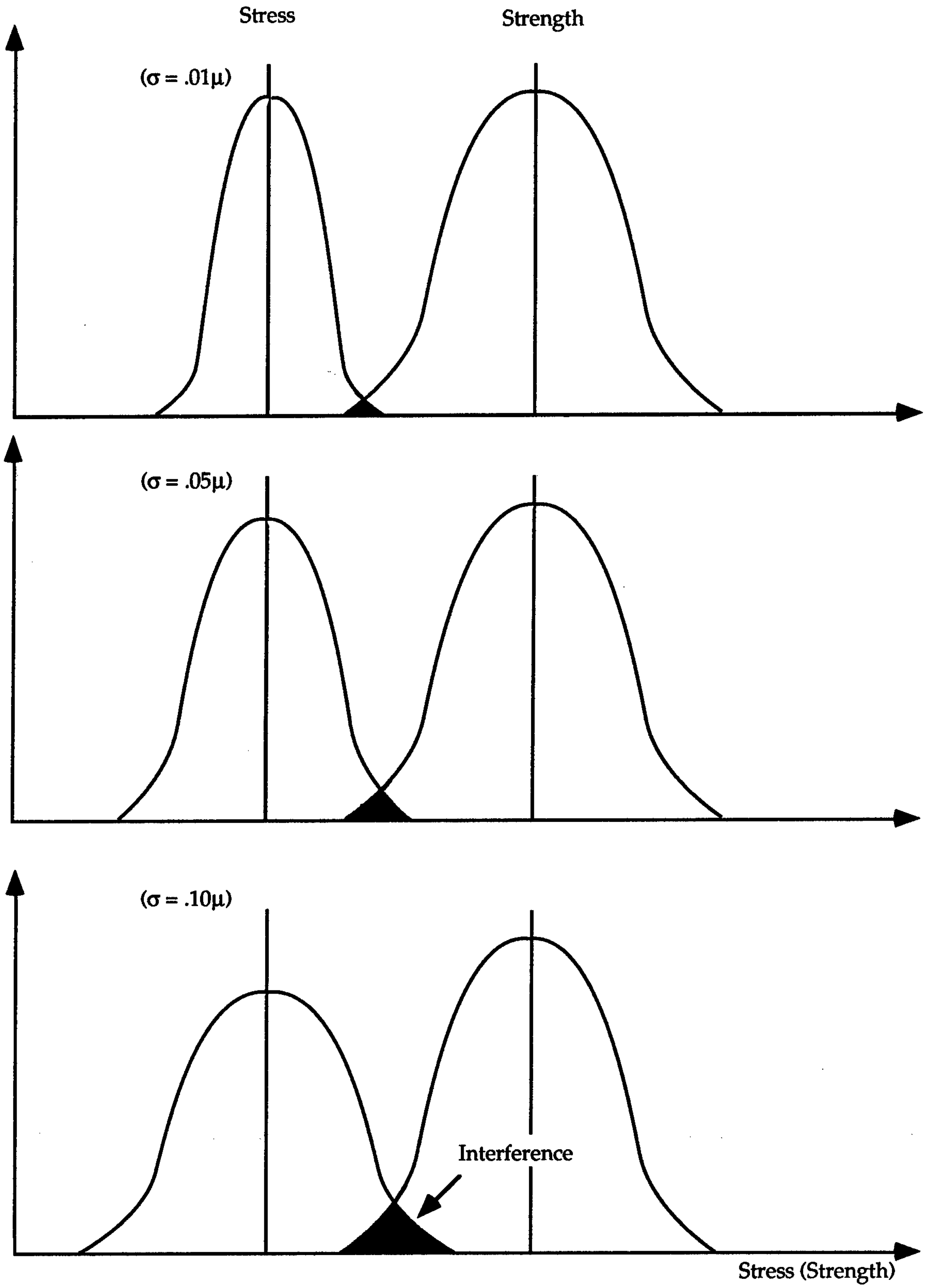


FIGURE 3.6-3: EFFECTS ON INTERFERENCE WITH CHANGES IN STANDARD DEVIATION (σ) OF THE STRESS DISTRIBUTION [42]

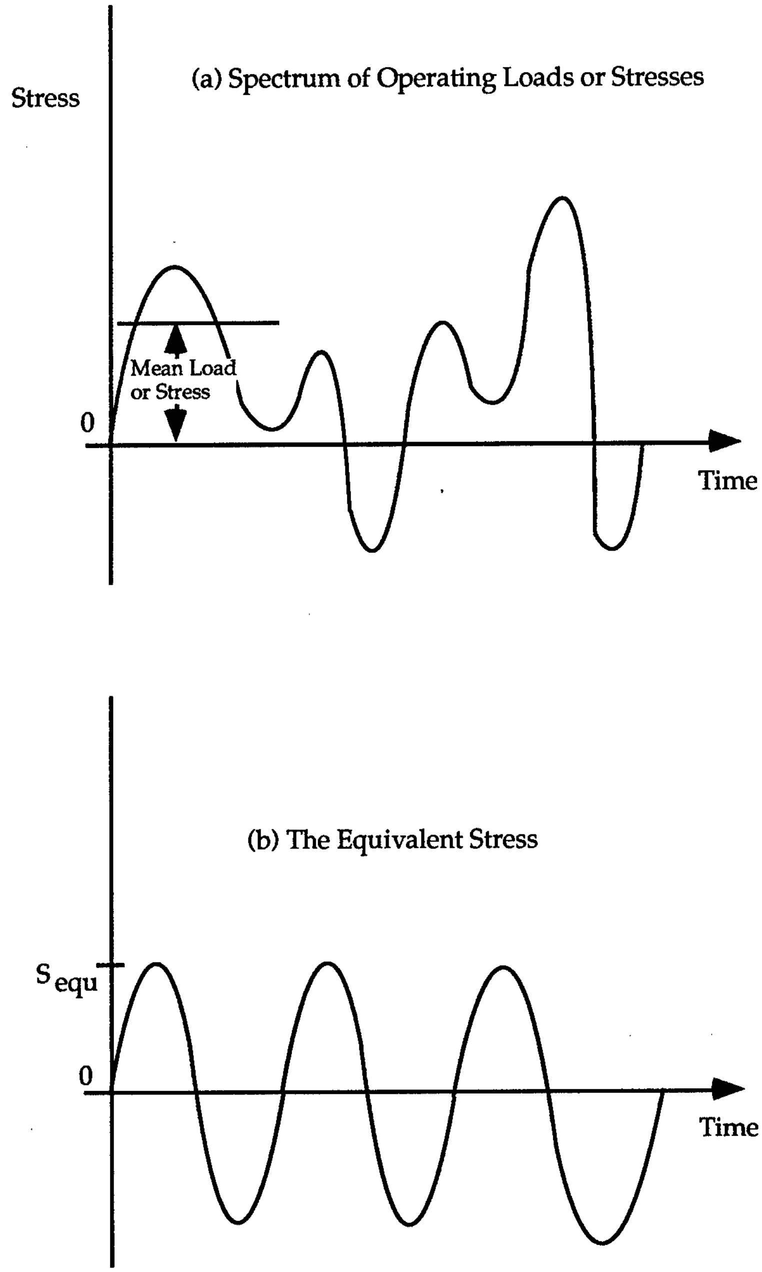


FIGURE 3.6-4: CONVERSION OF STRESS SPECTRUM TO EQUIVALENT STRESS [42]

Solution:

The appropriate table is found in RADC-TR-68-403 on page 182 (Code 37) which is reproduced in Table 3.6-1. The density function for hot rolled 2014-T6 is the Smallest Extreme-Value (S.E.V.), a two-parameter distribution. At 10^6 cycles, the parameters are:

$$\beta = .4523$$
$$M = 29.74$$

The stress level is given as $S_{equ} = 23.6$ ksi

Compute:

$$X = -\beta(S_{equ} - M)$$
$$X = -.4523\,(23.6 - 29.74) \quad (3\text{-}30)$$
$$X = 2.78$$

Entering the table on page 419 of RADC-TR-68-403 which is reproduced in Table 3.6-2 gives an "interference" of .0602 by interpolating for a value of X = 2.78. The reliability estimate is given by:

$$R = 1.0 - \text{Interference} \quad (3\text{-}31)$$
$$R = 1.0 - .0602$$
$$R = .9398$$

Case 2: Example

A cylindrical component is to be subjected in service to completely reversed bending stresses of ±23.6 ksi at ambient temperature. The design life is 10^6 cycles. Estimate the reliability if the component is constructed of aluminum alloy 7079-T652, forged, and mechanically polished.

TABLE 3.6-1: RADC-TR-68-403 ALLOY TABLE FOR 2014 ALUMINUM

S_u = 68 - 78 ksi
S_y = 60 - 71 ksi

2014 Aluminum

Rotary Beam Bending

Miscellaneous

Test Conditions						Fatigue Strength Distributions and Their Parameters										
								Life in Cycles								
Heat Treat.	Surf. Fin.	Freq.	Temp °F	K_t	Other	Dist.	Param.	1x10³	1x10⁴	5x10⁴	1x10⁵	5x10⁵	1x10⁶	5x10⁶	1x10⁷	1x10⁸
T-6 Wrought			80	1.0		Weibull	b	1.921*	1.781	1.682	1.636	1.770	1.522	1.642	1.689	1.824*
							θ	75.93	63.92	56.68	53.81	47.75	45.31	42.20	38.18	32.17
							x_o	68.38	57.85	51.46	48.94	43.23	41.34	36.55	34.66	29.07
T-3 H-Roll	M.P.		80	1.0		S.E.V.	β		.1986*	.2648	.2997	.3996	.4523	.6030	.6825*	1.030*
							M		67.72	50.79	44.88	33.66	29.74	22.30	19.70	13.06

(*) Extrapolated Values

TABLE 3.6-2: RADC-TR-68-403 INTERFERENCE TABLE

X	Interference F(X)	X	Interference F(X)
0.0	.6321	2.8	.0590
0.1	.5954	2.9	.0540
0.2	.5590	3.0	.0459
0.3	.5233	3.1	.0441
0.4	.4884	3.2	.0400
0.5	.4548	3.3	.0362
0.6	.4224	3.4	.0328
0.7	.3914	3.5	.0298
0.8	.3619	3.6	.0270
0.9	.3341	3.7	.0244
1.0	.3078	3.8	.0221
1.1	.2831	3.9	.0200
1.2	.2601	4.0	.0182
1.3	.2385	4.2	.0149
1.4	.2185	4.4	.0122
1.5	.2000	4.6	.0100
1.6	.1829	4.8	.0082
1.7	.1669	5.0	.0067
1.8	.1524	5.2	.0055
1.9	.1389	5.4	.0045
2.0	.1266	5.6	.0037
2.1	.1153	5.8	.0030
2.2	.1049	6.0	.0025
2.3	.0954	6.2	.0020
2.4	.0868	6.4	.0017
2.5	.0788	6.6	.0014
2.6	.0716	6.8	.0011
2.7	.0650		

Stress Distribution - Normal ($\sigma = 0$)
Strength Distribution - Smallest Extreme Value

Solution:

The appropriate table is found in RADC-TR-68-403 on page 233 (code 288) which is shown in Table 3.6-3. The density function for the fatigue strength is Weibull, a three-parameter distribution. At 10^6 cycles, the Weibull parameters are:

$b = 3.096$
$\theta = 28.96$
$x_0 = 20.51$

For this example, the standard deviation is assumed by engineering experience to be 5 percent of the mean; the actual standard deviation should be determined experimentally based on the particular situation when possible.

$S_{equ} = 23.6 \text{ ksi}$
$\sigma = .05(23.6) = 1.2 \text{ ksi}$

Compute:

$$A = \frac{x_o - S_{equ}}{\sigma} = \frac{20.51 - 23.6}{1.2} \qquad (3\text{-}32)$$

$A = -2.6$

and,

$$C = \frac{\theta - x_o}{\sigma} = \frac{28.96 - 20.51}{1.2} \qquad (3\text{-}33)$$

$C = 7.0$

and enter the tables in RADC-TR-68-403 on page 392 which is shown in Table 3.6-4 under b, A and C to find an estimate of interference.

Interference = .0688

The reliability is:

$R = 1.0 - .0688$
$R = .9312$

TABLE 3.6-3: RADC-TR-68-403 ALLOY TABLE FOR 7079 ALUMINUM

S_u = 70 - 79 ksi
S_y = 63 - 66 ksi

7079 Aluminum

Rotary Beam Bending

Miscellaneous

Test Conditions						Fatigue Strength Distributions and Their Parameters										
								Life in Cycles								
Heat Treat.	Surf. Fin.	Freq.	Temp °F	K_t	Other	Dist.	Param.	$1x10^3$	$1x10^4$	$5x10^4$	$1x10^5$	$5x10^5$	$1x10^6$	$5x10^6$	$1x10^7$	$1x10^8$
T-6			80	1.0	SP-3	S.E.V.	β	.2739*	.3010*	.3216	.3308	.3534	.3636	.3884	.3996	.4391*
							M	42.36	38.54	36.08	35.07	32.83	31.91	29.87	29.04	26.42
T-652	M.P. 16 RMS		80	1.0	SP-3	Weibull	b	3.113*	3.040	3.059	3.218	3.130	3.096	3.230	3.204*	3.132*
							θ	63.26	48.75	40.63	37.58	31.32	28.96	24.14	22.32	17.20
							x_o	44.73	34.71	28.88	26.30	22.11	20.51	16.87	15.64	12.14

(*) Extrapolated Values

TABLE 3.6-4: RADC-TR-68-403 INTERFERENCE TABLE

b = 3.00

$$C = \frac{\theta - x_o}{\sigma}, \quad A = \frac{x_o - \mu}{\sigma}$$

A	C = 1.00	C = 2.00	C = 3.00	C = 4.00	C = 5.00	C = 6.00	C = 7.00	C = 8.00	C = 9.00
.8	.0535	.0142	.0050	.0022	.0012	.0007	.0004	.0003	.0002
.6	.0777	.0218	.0078	.0035	.0018	.0011	.0007	.0005	.0003
.4	.1093	.0325	.0118	.0053	.0028	.0016	.0010	.0007	.0005
.2	.1493	.0471	.0176	.0080	.0042	.0025	.0016	.0011	.0007
.0	.1980	.0664	.0254	.0116	.0062	.0036	.0023	.0015	.0011
-.2	.2551	.0914	.0359	.0166	.0088	.0052	.0033	.0022	.0016
-.4	.3198	.1226	.0496	.0232	.0124	.0073	.0047	.0031	.0022
-.6	.308	.1605	.0670	.0318	.0171	.0101	.0064	.0043	.0031
-.8	.4651	.2051	.0888	.0426	.0230	.0137	.0087	.0059	.0042
-1.0	.5408	.2563	.1148	.0561	.0305	.0182	.0116	.0078	.0055
-1.4	.6854	.3750	.1819	.0922	.0510	.0306	.0197	.0133	.0094
-1.8	.8058	.5065	.2681	.1418	.0800	.0486	.0314	.0213	.0151
-2.2	.8930	.6369	.3702	.2057	.1189	.0731	.0475	.0324	.0230
-2.6	.9476	.7528	.4814	.2827	.1683	.1049	.0688	.0472	.0336
-3.0	.9773	.8452	.5936	.3703	.2278	.1446	.0958	.0661	.0473
-3.4	.9914	.9112	.6981	.4643	.2964	.1921	.1287	.0894	.0643
-3.8	.9971	.9536	.7884	.5596	.3721	.2469	.1678	.1175	.0849
-4.2	.9992	.9780	.8604	.6513	.4524	.3082	.2127	.1505	.1093
-4.6	.9998	.9905	.9137	.7346	.5340	.3744	.2630	.1881	.1377
-5.0	1.0000	.9963	.9501	.8064	.6139	.4439	.3180	.2302	.1698
-5.5	1.0000	.9990	.9772	.8775	.7067	.5323	.3917	.2886	.2153
-6.0	1.0000	.9998	.9907	.9280	.7876	.6188	.4687	.3521	.2661
-6.5	1.0000	1.0000	.9966	.9608	.8537	.6994	.5463	.4192	.3214
-7.0	1.0000	1.0000	.9989	.9804	.9045	.7713	.6219	.4882	.3803
-8.0	1.0000	1.0000	.9999	.9962	.9658	.8820	.7575	.6242	.5038
-9.0	1.0000	1.0000	1.0000	.9995	.9904	.9485	.8614	.7458	.6257
10.0	1.0000	1.0000	1.0000	1.0000	.9979	.9813	.9303	.8430	.7359

Stress Distribution: Normal

Strength Distribution: Weibull

Case 3: Example

A design for an aircraft part specifies the following:

Material:	2024 aluminum (S_u = 70 ksi, S_y = 50 ksi)
Design Life:	10^6 cycles
Type of Loading:	Axial
Size:	.125" sheet
Surface Finish:	Electropolished
Stress Concentration Factor:	K_t = 1.5 (Milled edge)
Operating Temperature:	Room Temperature

The first step is to determine the strength distribution parameters corresponding to the design conditions. The strength distribution at 10^6 cycles is found in the alloy tables in RADC-TR-68-403 on page 194 (Code No. 102) which is shown in Table 3.6-5. The strength distribution is the smallest extreme value and the parameters are:

β = .8348
M = 19.98 ksi

In order to determine the stress distribution parameters ($\sigma = S_{equ}$ and $K = \mu$), a prototype of this aircraft part was instrumented and the stress spectrums were recorded as shown in Columns 1 and 2 of Table 3.6-6.

The first step to determine the equivalent stress $\left(S_{equ}\right)$ is to apply Miner's rule[4] to determine the equivalent number of cycles to failure $\left(N_{equ}\right)$ from the stress spectrum data in Table 3.6-6. Miner's rule, also known as the "Palmgren-Miner cycle-ratio summary theory," analyzes cumulative fatigue damage. This theory states that at the point of failure,

$$\sum \frac{n_i}{N_i} = C \tag{3-34}$$

4 Miner, M.A., "Cumulative Damage In Fatigue." Journal of Applied Mechanics, Volume 12, 1945.

TABLE 3.6-5: RADC-TR-68-403 ALLOY TABLE FOR 2024 ALUMINUM

S_u = 61 - 81 ksi
S_y = 45 - 61 ksi

2024 Aluminum
(Sheet or Plates)

Axial (Completely Reversed)
Effects of Stress Concentration

Heat Treat.	Surf. Fin.	Freq.	Temp °F	K_t	Other		Param.	$1x10^3$	$1x10^4$	$5x10^4$	$1x10^5$	$5x10^5$	$1x10^6$	$5x10^6$	$1x10^7$	$1x10^8$
T-3	Electropolished	37	80	2.8	SP-3	Normal	σ	4.820*	3.707	3.086	2.851	2.373	2.193	1.825	1.687	1.297*
							μ	31.88	24.52	20.41	18.86	15.70	14.50	12.07	11.15	8.583
T-3	Electropolished	37	80	2.6	SP-3	Weibull	b	1.781*	1.862	1.775	1.977	1.916	1.893	1.863	1.863	1.863*
							θ	35.07	26.37	21.59	19.83	16.24	14.90	12.20	11.20	8.423
							x_o	27.76	20.74	17.10	15.45	12.72	11.69	9.604	8.813	6.625
T-3	Electropolished	24-30	80	2.4	SP-3	Weibull	b	1.984*	1.767	1.980	1.936	1.844	1.808	1.734	1.719	1.719*
							θ	36.25	27.76	23.07	21.29	17.67	16.31	13.54	12.49	9.579
							x_o	30.29	23.48	19.28	17.84	14.88	13.76	11.47	10.59	8.123
T-3	Electropolished	24-30	80	2.2	SP-3	Normal	σ	3.561*	2.924*	2.547	2.401	2.092	1.971	1.717	1.618	1.329*
							μ	27.98	22.97	20.01	18.86	16.43	15.49	13.49	12.71	10.44
T-3	Milled Edges Electropolish	25	80	1.5	SP-4	S.E.V.	β	.3392*	.4580	.5649	.6183	.7627	.8348	1.029*	1.127*	1.521*
							M	49.18	36.43	29.53	26.98	21.87	19.98	16.20	14.80	10.96
T-3	Milled Edges Electropolish	25	80	2.0	SP-4	Weibull	b	1.623*	1.907	1.789	1.807	1.872	1.785*	1.908*	1.852*	1.722*
							θ	43.97	28.74	21.33	18.76	13.93	12.25	9.100	8.003	5.224
							x_o	37.15	23.86	17.84	15.67	11.59	10.25	7.555	6.668	4.388

(*) Extrapolated Values

TABLE 3.6-6: STRESS AND LIFE DATA FOR MINER'S RULE

Stress Spectrum		**Miner's Rule Data**	
Completely* Reversed Axial Stresses, ksi	**Number of Occurrences, n_i**	**Cycles to Failure, N_i**	$\frac{n_i}{N_i}$
1	**2**	**3**	**4**
11.7	240	5.1×10^7	4.70×10^{-6}
12.5	217	3.0×10^7	7.24×10^{-6}
13.0	176	2.1×10^7	8.39×10^{-6}
13.8	150	1.3×10^7	11.52×10^{-6}
14.1	110	1.1×10^7	10.00×10^{-6}
14.9	75	7.2×10^6	10.40×10^{-6}
15.7	52	4.8×10^6	10.81×10^{-6}
15.9	20	4.4×10^6	4.55×10^{-6}
16.0	5	4.2×10^6	1.19×10^{-6}
	$\Sigma n_i = 1035$	$\Sigma \frac{n_i}{N_i} = 68.80 \times 10^{-6}$	

*Actually, the stress was not completely reversed. It was reduced with the aid of a Goodman diagram to a completely reversed stress.

where,

n_i = number of cycles of stress (s)

N_i = life to failure corresponding to stress (s)

C = a constant determined experimentally (usually found in the range $0.7 \leq C \leq 2.2$ but many authorities recommend using 1.0)

The number of cycles to failure (N_i) corresponding to the stresses in Column 1 of Table 3.6-6 are determined from the S-N curves of the material shown in Figure 3.6-5. The results are recorded in Column 3 of Table 3.6-6.

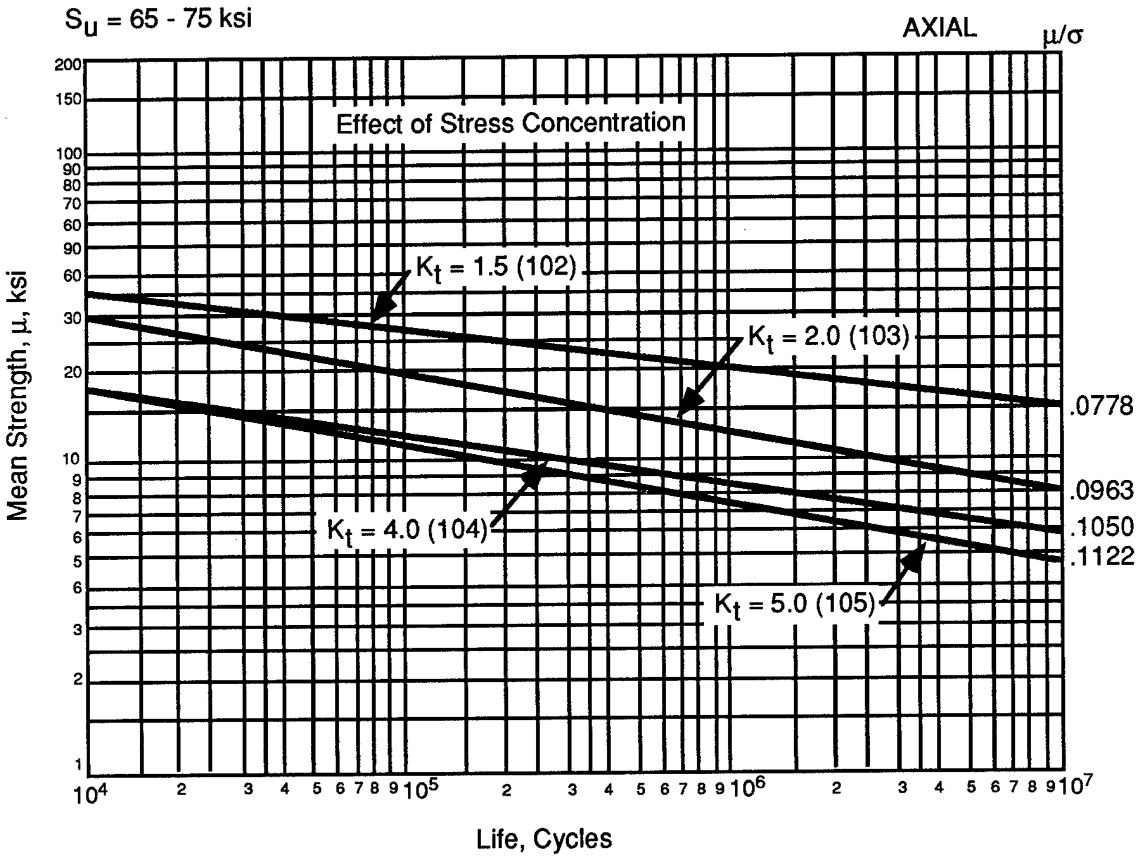


FIGURE 3.6-5: S-N DIAGRAM FOR 2024 ALUMINUM (SHEETS)

Using Miner's Rule, Equation (3-34), and the tabulated data in Table 3.6-6, an equivalent number of cycles to failure $\left(N_{equ}\right)$ is determined as follows:

$$N_{equ} = \frac{\Sigma n_i}{\Sigma \frac{n_i}{N_i}} \tag{3-35}$$

$$N_{equ} = \frac{1035}{68.80 \times 10^{-6}} = 1.51 \times 10^{7} \text{ cycles}$$

From the S-N curve (Figure 3.6-5) the stress corresponding to $N_{equ} = 1.51 \times 10^7$ cycles was extrapolated to approximately:

$$S_{equ} = 13.5 \text{ ksi}$$

In some engineering applications, the scatter in operating stresses is very small. Therefore, in these applications the standard deviation of the equivalent stress can be assumed to be zero. In those engineering applications where the scatter in stress is appreciable, the standard deviation typically lies in the range,

$0.01\,\mu \leq \sigma \leq 0.10\,\mu,$ where μ is the mean stress

In the absence of any specific information, an average value of $\sigma = .05\mu$ can be assumed as an approximation. For the present problem, interference will be calculated for the two cases: $\sigma = 0$ and $\sigma = .05\mu$.

Thus, the stress parameters are:

1. $\mu = S_{equ} = 13.5$ ksi and $\sigma = 0$
2. $\mu = 13.5$ ksi and $\sigma = .05\mu = .05\,(13.5) = .675$ ksi

Once the strength and stress distribution parameters are determined, the interference and the reliability (Reliability = 1.0 - Interference) can be determined.

For the case where stress is normally distributed with $\mu = S_{equ}$ and $\sigma = 0$ and a Smallest Extreme-Value distribution of strength, the interference can be determined from:

$$S_{equ} = 13.50 \text{ ksi}$$

$$\beta = .8348$$

$$M = 19.98$$

Compute:

$$X = -\beta(S_{equ} - M) \qquad (3\text{-}36)$$

$$X = -.8348\,(13.50 - 19.98)$$

$$X = 5.404$$

From the interference table contained in RADC-TR-68-403 on page 419 which is shown in Table 3.6-7, the interference is determined to be .0045. The reliability is then computed as:

$$R = 1 - \text{Interference} \tag{3-37}$$

$$R = 1 - .0045$$

$$R = .9955$$

For cases where stress is normally distributed with $\sigma \neq 0$ and strength is distributed according to the Smallest Extreme-Value (S.E.V.), the interference is determined as follows:

Strength (S.E.V.)	Stress (Normal)
$\beta = .8348$	$\mu = S_{equ} = 13.50$ ksi
$M = 19.98$ ksi	$\sigma = .05\mu = .675$ ksi

Compute:

$$\alpha = \beta\sigma = (.8348)(.675) = .564 \tag{3-38}$$

and,

$$\gamma = \beta\,(S_{equ} - M) = (.8348)(13.50 - 19.98) = -5.4$$

and from the table on page 422 of RADC-TR-68-403, which is shown in Table 3.6-8, the interference value corresponding to these parameters is determined using linear interpolation to be .0059. The corresponding component reliability is calculated to be:

$$R = 1 - \text{Interference} \tag{3-39}$$

$$R = 1 - .0059$$

$$R = .9941$$

TABLE 3.6-7: RADC-TR-68-403 INTERFERENCE TABLE

X	Interference F(X)	X	Interference F(X)
0.0	.6321	2.8	.0590
0.1	.5954	2.9	.0540
0.2	.5590	3.0	.0459
0.3	.5233	3.1	.0441
0.4	.4884	3.2	.0400
0.5	.4548	3.3	.0362
0.6	.4224	3.4	.0328
0.7	.3914	3.5	.0298
0.8	.3619	3.6	.0270
0.9	.3341	3.7	.0244
1.0	.3078	3.8	.0221
1.1	.2831	3.9	.0200
1.2	.2601	4.0	.0182
1.3	.2385	4.2	.0149
1.4	.2185	4.4	.0122
1.5	.2000	4.6	.0100
1.6	.1829	4.8	.0082
1.7	.1669	5.0	.0067
1.8	.1524	5.2	.0055
1.9	.1389	5.4	.0045
2.0	.1266	5.6	.0037
2.1	.1153	5.8	.0030
2.2	.1049	6.0	.0025
2.3	.0954	6.2	.0020
2.4	.0868	6.4	.0017
2.5	.0788	6.6	.0014
2.6	.0716	6.8	.0011
2.7	.0650		

Stress Distribution: Normal ($\sigma = 0$)
Strength Distribution: Smallest Extreme Value

TABLE 3.6-8: RADC-TR-68-403 INTERFERENCE TABLE

γ	α 0.500	0.600	0.700	0.800	0.900	1.000	$\alpha = \beta\sigma$, $\gamma = \beta(\mu - M)$ 1.100	1.200	1.300	1.400
0.	.6301	.6286	.6265	.6240	.6213	.6182	.6151	.6119	.6086	.6054
-1.	.3263	.3333	.3409	.3487	.3565	.3641	.3714	.3782	.3847	.3907
-2.	.1394	.1451	.1517	.1592	.1674	.1761	.1853	.1948	.2043	.2139
-3.	.0544	.0572	.0605	.0644	.0691	.0743	.0802	.0868	.0938	.1013
-4.	.0205	.0216	.0230	.0245	.0267	.0290	.0318	.0351	.0388	.0430
-5.	.0076	.0080	.0086	.0092	.0100	.0110	.0121	.0135	.0151	.0170
-6.	.0028	.0030	.0032	.0034	.0037	.0041	.0045	.0050	.0057	.0065
-7.	.0010	.0011	.0012	.0013	.0014	.0015	.0017	.0019	.0021	.0024
-8.	.0004	.0004	.0004	.0005	.0005	.0006	.0006	.0007	.0008	.0009
-9.	.0001	.0002	.0002	.0002	.0002	.0002	.0002	.0003	.0003	.0003
-10.	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-11.	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

For $\alpha = 0$, Interference $= 1 - e^{-e^{\gamma}}$

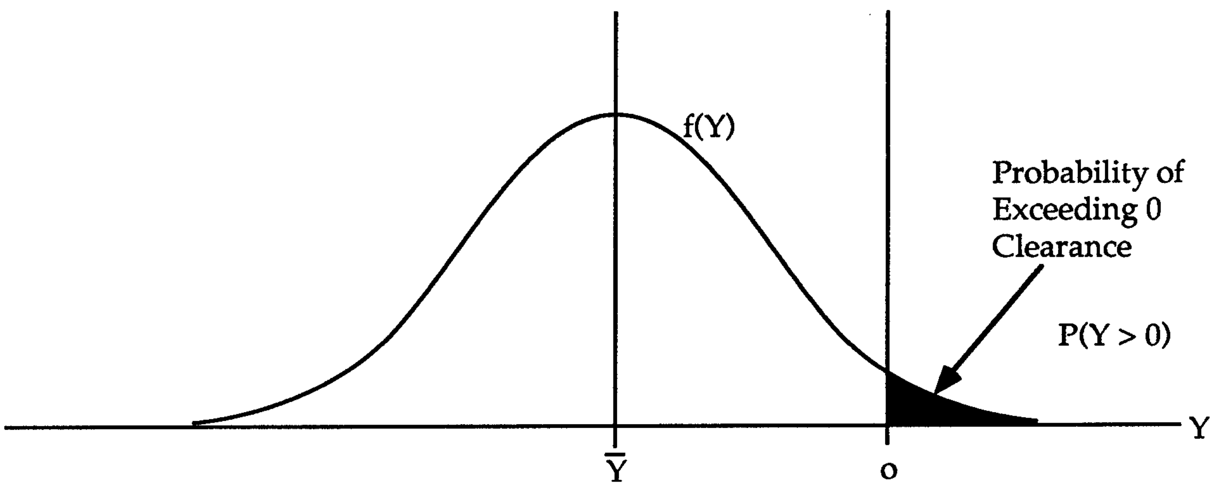
Stress Distribution: Normal

Strength Distribution: Smallest Extreme Value

3.6.2 Using Interference Analysis to Evaluate Part Geometry

Component dimensions such as lengths, widths, thickness and other physical features comprise one class of variables of prime importance in mechanical design. Dimensionally described geometry and especially random geometric variations created by the production process, impose sizable direct influences on the probability of failure of components and systems. Statistical dimensional descriptions and analyses, like the one that follows, provide an opportunity to account for machining and processing variability in the design process itself and estimate the impact of dimensional variability on the performance of a population of like mechanical components.

The method of interference theory (refer to Figure 3.6-6) was employed to evaluate the probability of losing an interference fit strictly based on the expected geometric variations in a stator assembly. The variables shown in Figure 3.6-7 were used to identify the inside and outside unassembled diameters for the housing, intermediate ring and lamination stack, which were applied to this evaluation. Table 3.6-9 identifies the dimensions used in the analysis.



$$\text{Probability of Maintaining Interference Fit} = 1 - \int_0^\infty f(Y)dY$$

$$\text{Probability of Exceeding 0 Clearance} = \int_0^\infty f(Y)dY$$

FIGURE 3.6-6: INTERFERENCE THEORY AS APPLIED TO EVALUATING THE PROBABILITY OF LOSING/MAINTAINING INTERFERENCE FIT

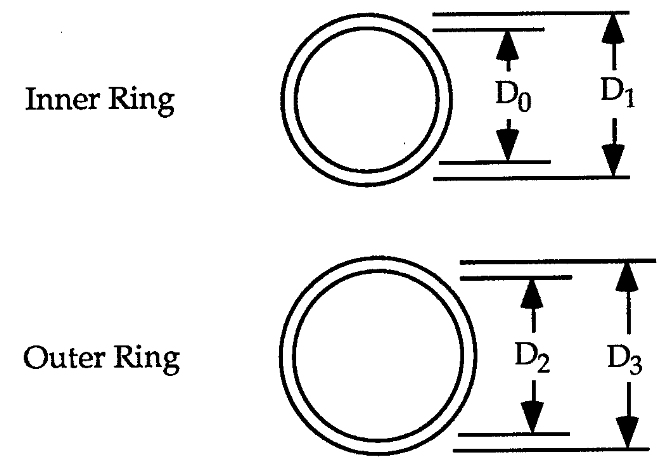


FIGURE 3.6-7: COMPONENT GEOMETRY VARIABLES

TABLE 3.6-9: RING GEOMETRY

Component Geometry Variable	Description	Unassembled Dimensional Tolerances (Inches)	Mean Value*, $\bar{D}_i$ (Inches)	Standard Deviation* σ_i (Inches)
D_0	Inside Diameter, Inner Ring	3.550/3.530	3.54	.0033
D_1	Outside Diameter, Inner Ring	3.8225/3.8220	3.82225	.000083
D_2	Inside Diameter, Outer Ring	3.82266/3.82166	3.82216	.000167
D_3	Outside Diameter, Outer Ring	3.9435/3.9425	3.9430	.00016

*Assumed Normal Distribution

The following model was then established to evaluate the interference fit (refer to Figure 3.6-6 and Table 3.6-9):

$$Y = D_2 - D_1 \tag{3-40}$$

Note: Y is a new variable that must be negative to maintain an interference fit.

From the algebra of expectation of random variables (Reference: "Probabilistic Mechanical Design", by E.B. Haugen), the following equations can be applied to calculate the mean and standard deviation for the new variable, Y.

$$\overline{Y}_1 = \overline{D}_2 - \overline{D}_1 \tag{3-41A}$$

$$\sigma_Y = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{3-41B}$$

Substituting the values of $\overline{D}_1$ and $\overline{D}_2$ into Equation (3-41A) yields:

$$\overline{Y} = \overline{D}_2 - \overline{D}_1$$

$$\overline{Y} = 3.82216 - 3.82225$$

$$\overline{Y} = -.00009$$

Substitution of the values of σ_1 and σ_2 into Equation (3-41B) yields:

$$\sigma_Y = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$\sigma_Y = \sqrt{.00083^2 + .000167^2}$$

$$\sigma_Y = .0001865$$

therefore,

$$\left(\overline{Y}, \sigma_Y\right) = (-.00009, .0001865)$$

The probability of maintaining an interference fit (P) is shown graphically in Figure 3.6-6 and is given by the following equation:

$$P = 1 - \int_0^{\infty} f(Y)\, dY \tag{3-42}$$

Now, assuming the standard normal distribution, the following equation results:

$$P = 1 - \int_a^b \frac{e^{-u^2/2}}{\sqrt{2\pi}}\, du \tag{3-43}$$

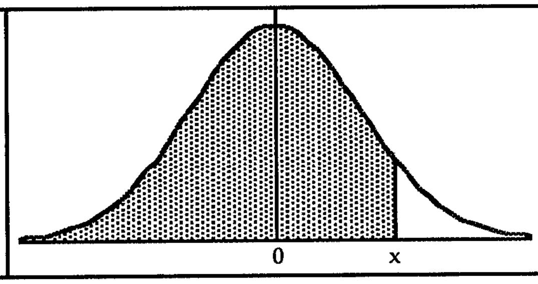
The limits of integration associated with the standard normal distribution are calculated and the probability of maintaining an interference fit is determined using Table 3.6-10.

TABLE 3.6-10: AREAS UNDER THE STANDARD NORMAL CURVE

Areas Under the Standard Normal Curve

from $-\infty$ to x

$$\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$



x	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5754
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7258	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7518	.7549
0.7	.7580	.7612	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7996	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The calculations are as follows:

For variable Y: $\left(\overline{Y}, \sigma_Y\right)$ = (-.00009, .0001865), calculate the lower limit of integration 'a' at Y = 0 as follows:

$$a = \frac{Y - \overline{Y}}{\sigma_Y} = \frac{0 - (-.00009)}{.0001865}$$

$$a = .48$$

Calculate upper limit 'b' at Y = ∞ as follows:

$$b = \frac{Y - \overline{Y}}{\sigma_Y} = \frac{\infty - (-.00009)}{.0001865}$$

$$b = \infty$$

Now, substitute the new limits of integration into Equation (3-43) and utilize Table 3.6-10 to evaluate the integral.

$$P = 1 - \int_{.48}^{\infty} \frac{e^{-u^2/2}}{\sqrt{2\pi}}\, du$$

$$P = .6844$$

The sensitivity of the current component geometry to potential failure in terms of losing interference fit is based on a comparison of the lower limits of integration associates with the standard normal distribution which is correlated to the probability of failure. As this lower limit of integration drops below 3.9, the probability of maintaining the interference fit decreases.

3.7 Evaluating Part Reliability Using Surrogate Data Sources

Surrogate data sources provide estimates of reliability numerics for many generic part types. These data sources typically present data in the form of failures/hour, failures/cycle or failures/mile. Data is generally collected from a wide range of part applications and operating stress profiles and grouped together based on similar part types and similar application environment.

Currently, most mechanical reliability data bases find it necessary to total the number of observed failures and part hours. Computation of a failure rate requires the analyst to accept the assumption that failures of mechanical parts follow the exponential distribution and display a constant failure rate. This assumption is necessary due to the virtual absence of data containing individual times or cycles to failure.

A typical data set may include the following information:

Total Operating Time = 45,875 hours
Total Failures = 7

Given this information and assuming the exponential failure model, a constant hazard rate is calculated:

$$\text{Failure Rate} = \frac{7}{45{,}875} = 0.000153 \text{ failures/hour}$$

If the actual time to failure data was available, it would be much more appropriate to use Weibull Analysis. But in the majority of cases the time to failure is unknown. It is more often the case that what is known is the total number of failures and total operating time.

Surrogate data sources almost invariably represent data for a variety of similar components. In the above example, it may have been assumed that all parts were of the same make and model. Often enough data can not be collected for a particular piece part and must be combined with data from similar parts. Failure rates presented in surrogate data sources may be the combination of several different parts of similar design and function.

The application of the parts and the parts chosen for the application can have a great affect on the failure rates. A bearing used in a fighter aircraft will show different failure characteristics than that of the same bearing used in a stationary low production milling machine. For this reason, it is necessary for surrogate data sources to separate failure information based on the operational profile. Some sources have twelve or more different environment profiles. This attempts to account for different stress profiles exhibited in various applications and also to differentiate between parts chosen for a particular application.

There are a variety of shortcomings with surrogate data sources that are worth noting. The underlying failure distribution is typically assumed to be exponential and constant failure rates are presented. This is done for mathematical simplicity and ease of data collection. For mechanical parts, the exponential distribution assumption may not be the most appropriate selection. These components generally show an increased probability of failure as operating time increases due to wear, fatigue, corrosion, etc. Finally, and perhaps the most over looked short coming of many surrogate data sources, is the lack of a detailed description of the parts that comprise the constant failure rate. Some generic reliability databases have worked to improve the level of detail relating to their reliability numerics but much work needs to be done to improve the quality of mechanical surrogate data sources. Some of the more popular surrogate data sources will be discussed in this section.

3.7.1 Nonelectronic Parts Reliability Data 1991 (NPRD-91)

NPRD-91 provides a comprehensive source of constant failure rate data for over 1,400 different part types. NPRD-91 is developed by the Reliability Analysis Center and represents a summary of the RAC nonelectronic database. RAC has been compiling this database since 1970 and typically acquires data from sources such as:

- Published reports and papers
- Data collected from government sponsored studies
- Data collected from military systems
- Data collected from commercial systems
- Data submitted directly to RAC

NPRD-91 provides "part summaries" in Section 2.0 and "part details" in Section 3.0. An example of the data presented in the part summary is shown in Figure 3.7-1 for a Ni-Cd rechargeable battery.

2-10 Part Summaries							NPRD-91
Part Description	**Qual Lev**	**App Env**	**Data Source**	**Fail Per E6 Hours**	**Total Failed**	**Operating Hours (E6)**	**Detail Page**
Battery, Rechargeable, Ni-Cd				0.5197			
	Com	GF	NPRD-010	0.4305	342	794.3832	3-6
	Mil			0.5452			
		AIF	25100-000	6913.2118	511	0.0739	3-6
		SF		0.0234			
			10219-034	0.1320	8	60.5910	3-6
			23020-000	<0.0236	0	42.3975	3-6
			NPRD-016	0.0011	8	7438.0000	3-6
			NPRD-120	0.0920	4	43.4655	3-6

FIGURE 3.7-1: NPRD-91 PART SUMMARY EXAMPLE

The data fields provided in the part summary listing shown in Figure 3.7-1 have the following definitions:

- Part Description — Description of part including the major family of parts and specific part type breakdown within the part family.

- Qual Lev — The Quality Level of the part as indicated by:

  Mil-Parts procured in accordance with MIL specifications.
  Com - Commercial quality parts.
  Unk (Unknown) - Data resulting from a device of unknown quality level

- App Env — The Application Environment describes the conditions of field operation. These environments are consistent with MIL-HDBK-217. In some cases, environments more generic that those used in MIL-HDBK-217 are used. For example: "A" indicates the part was used in an Airborne environment, but the precise location and aircraft type was not known. Environments proceeded by the term "No" are indicative of non-operating systems in the specified environment.

- Data Source — Source of data comprising this entry. The source number may be used as a reference to review individual data source descriptions.

- Failure Rate — For individual data entries (same part type, environment, quality, and source), this is the total number of failures divided by the total number of operating hours. For roll-up data entries (i.e., those without sources listed) failure rate is derived using the data merge algorithm described in this section. A failure rate preceded by a "<" is representative of entries with no failures. The failure rate listed was calculated by using a single failure divided by the given number of operating hours. The resulting number is a worst case failure rate and the real failure rate is less than this value. All failure rates are presented in a fixed format of four decimal places after the decimal point. The user is cautioned that data presented has inherently high variability and that four decimal places does not imply any level of precision or accuracy.

- Total Failed — The total number of failures observed in the merged data records.
- Operating Hours (E6) — The total number of operating hours observed in merged data records presented in millions of hours.
- Detail Page — The NPRD-91 page number containing the detail data which comprise the summary record.

The detailed section can be accessed as indicated by the "Detailed Page" field to acquire further design information (if available). An example of the part detail is provided in Figure 3.7-2 for the Ni-Cd battery.

3-6 Part Details **NPRD-91**

Part Desc.	Qual Lev	App Env	Data Source	Part Characteristics	Fail/Hours (E6)
Battery, Rechargeable, Ni-Cd					
	Com	GF	NPRD-010	-No Details, Pop:23568	136/61.2768
				-# Cells:3, Pop:55,	0/0.1430
				-# Cells:3, Pop:935,	8/2.4310
				-# Cells:21, Pop:44,	0/0.1144
				-# Cells:6, Pop:467,	3/1.2142
				-# Cells:4, Pop:8387,	13/21.8062
				-# Cells:1, Pop:262151,	171/681.5926
				-# Cells:20, Pop:1628,	3/4.2328
				-# Cells:10, Pop:3033,	8/7.8858
				-# Cells:8, Pop:5264,	0/13.6864
	Mil	AIF	25100-000	-No Details,	511/0.0739
	Mil	SF	10219-034	-No Details, Pop:3370	8/60.5910
	Mil	SF	23020-000	-No Details,	0/42.3975
	Mil	SF	NPRD-016	-No Details,	8/7438.0000

FIGURE 3.7-2: NPRD-91 PART DETAIL EXAMPLE

In summary, NPRD-91 provides historical reliability data on a wide variety of part types and aids engineers in establishing the relative reliability of various part types.

3.7.2 RADC Nonelectronic Reliability Notebook

The Nonelectronic Reliability Notebook is the result of research conducted at Hughes Aircraft Company, Ground System Group, Fullerton, California for Rome Air Development Center, currently Rome Laboratory. The purpose of the notebook is to serve as a reference document for the reliability characteristics of the most commonly used nonelectronic parts. This intent is similar to the purpose of NPRD-91. The notebook also presents useful reliability and life data analysis methods applicable to nonelectronic parts such as discussions of:

- Statistical failure models
- Design of statistical experiments
- Estimation methods
- Interference analysis

The reference section of this contains point estimates of the failure rates of the parts covered. In the majority of cases, the nonelectronic parts covered in the Notebook are adequately described in the reliability sense by a constant failure rate. When the part does not experience a constant failure rate, a Weibull analysis is presented where failure times were available. Data was screened to exclude secondary failures and failures caused by maintenance personnel.

An example of the data contained in the Nonelectronic Reliability Notebook is shown below.

Pump / Vacuum - Lobe Type				Identification Number 179	
			Failure Rate (Failures Per Million Hours)		
Env	Dist. Type	Mean Estimate (Hours)	80% Lower Bound	Failure Rate Estimate	80% Upper Bound
GF	Exponential	4091	199.898	244.444	298.946

Env	Number of Sources	Number of Parts Failed	Total Part Operating Hours	Comments
GF	1	22	90000	

Pump / Vacuum - Ring Seal Type				Identification Number 180	
			Failure Rate (Failures Per Million Hours)		
Env	Dist. Type	Mean Estimate (Hours)	80% Lower Bound	Failure Rate Estimate	80% Upper Bound
GF	Exponential	90000	2.480	11.111	33.275

Env	Number of Sources	Number of Parts Failed	Total Part Operating Hours	Comments
GF	1	1	90000	

3.7.3 IEEE-STD-500 Reliability Data

The IEEE 500 Reliability Data book is a guide which is intended to establish a consistent method for collecting and summarizing reliability information for electrical, electronic, sensing components and mechanical equipment that is used in the nuclear industry. The data contained is intended for use in reliability models developed by designers and analysts.

For the purpose of this edition, (IEEE-500-1984), a weighted geometric mean was chosen as the maximum likelihood estimator of the failure rate. The maximum likelihood estimator was determined by the following modification of the geometric means as described in "A Statistical Model for Combining Biased Expert Opinions", Los Alamos Report LA-UR-83-531 by H.F. Martz and M.C. Bryson.

$$\hat{\theta} = \prod_{i=1}^{n} \hat{\theta}_i W_i \qquad (3\text{-}44)$$

where,

$\hat{\theta}$ = the maximum likelihood of failure rates or outage times
i = data source number, 1,2,3 ......
W_i = a dispersion dependent weighting factor for each data source

where,

$$W_i = \frac{\tau_i^{-2}}{\sum_{i-1}^{\hat{n}} \tau_i^{-2}}$$

τ_i^{-2} = reciprocal of the variance of the log of each data source

The data is presented in a format where a failure rate is given with the high and low values representing the range and a recommended value (REC) as a best estimate. Two types of failure rates are presented where applicable; failures per 10^6 hours and failures per 10^6 cycles. Outage times are also indicated as well as the failure rate for a particular failure mode. An example of the data contained in IEEE-500-1984 is shown below.

(Composite of Ref 1, 3, and 7.1.1.X) CHAPTER: 7 Valve Operators	SECTION:		7.1 Electric			SUBSECTION: 7.1.1 Motors						
ITEM OR EQUIPMENT DESCRIPTION												
Failure Mode	Failure Rate								(*) Out of Service (†) Repair Time or ( ) Restore (Hours)			
	Failures/10^6 Hours				Failures/10^6 Cycles **							
	Low	Rec	High	Ref	Low	Rec	High	Ref	Low	Rec	High	Ref
ALL MODES	0.01	0.63	500		0.72	4.87	50.0		1.0*	41.50	6.49E3	
CATASTROPHIC	0.004	0.25	200.0		0.29	1.95	20.0					
Spurious Opening	0.002	0.13	100.0		0.15	0.98	10.0					
Spurious Closing	0.002	0.12	100.0		0.14	0.97	10.0					
DEGRADED	0.006	0.38	300.0		0.43	2.92	30.0					
Partially Opening					0.17	1.17	12.0					
Partially Closing					0.26	1.75	18.0					
**One Cycle = One Demand												

SECTION D

SYSTEM RELIABILITY ENGINEERING

4.0 EVALUATING THE RELIABILITY OF SYSTEMS

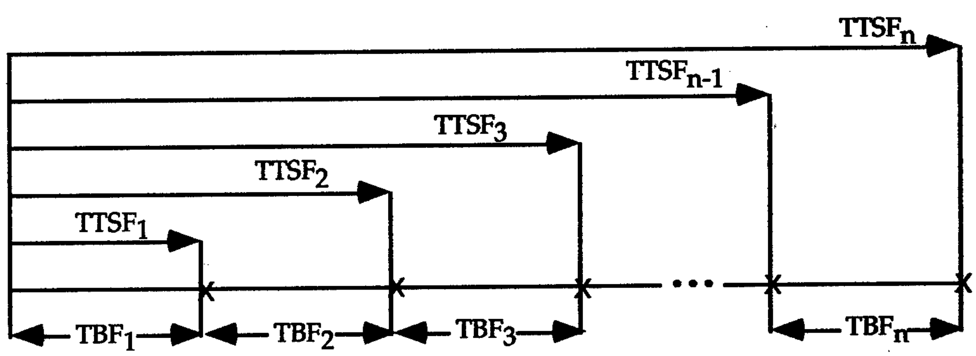
4.1 Concept of a Point Process

Evaluating the reliability of systems begins with the realization that a sequential failure process exists for the system. This failure process is composed of many sequential random variables.

This system failure process is depicted in Figure 4.1-1. A point process is characterized by observations in the form of point events occurring in a continuum such as time. Such processes arise in many fields of study such as economics, physics and system reliability. A point process can be defined by specifying:

1) description of each event and the measure of time (e.g., operating hours rounds, cycles, etc.)

2) the observed intervals between successive events denoted TBF_1, TBF_2, ... TBF_N or the instants of occurrence of the events measured from the time origin denoted $TTSF_1$, $TTSF_2$, $TTSF_3$, ... $TTSF_N$



$TTSF_i$ = Failure arrival times for the system
TBF_i = Interarrival times or time between (successive) failure

FIGURE 4.1-1: REPAIRABLE SYSTEM FAILURE PROCESS

The observed intervals between successive events (TBF_1, TBF_2, ...) are termed interarrival times and the intervals to the occurrence of events measured from the time origin ($TTSF_1$, $TTSF_2$, ...) are termed arrival times. The arrival times are obtained by forming the cumulative sums of the interarrival times or

$$TTSF_1 = TBF_1,\ TTSF_2 = TTSF_1 + TBF_2,\ TTSF_3 = TTSF_2 + TBF_3, \ldots \quad TTSF_n = TTSF_{n-1} + TBF_n \tag{4-1}$$

where,

$TTSF_n$ = is the arrival time of the nth event

Given that a system can be characterized by a point process, a major concern for the reliability analyst lies in describing this detailed pattern of occurrence. Of particular concern is whether a trend or some other systematic feature exists. For example, trends indicating that the interarrival times (TBF_i) are becoming smaller over a period of observation indicates that system performance is deteriorating. The modeling and analysis of point processes provides measures to quantify such systems.

Unlike part failure data, the chronological ordering of time-between-failure (TBF) data is extremely important for a repairable system. Disrupting or failing to track this ordering of failure events results in wasted effort! This can be illustrated in the following example, given the following three time-between-failure (TBF) values of: 10, 50 and 100. If the sequential order of these events is unknown, then a total of six different unique system processes can be created. The total number of unique system processes can be calculated using the equation for permutations:

$$P_r^n = \frac{n!}{(n-r)!} \tag{4-2}$$

where,

n = total number of objects
r = number of objects selected out of the total number

Substituting n = 3 and r = 3, equation (4-2) yields:

$$P_r^n = \frac{3!}{(3-3)!} = \frac{6}{1} = 6$$

The six *unique* system processes, identified by their unique arrangement of interarrival values, are as follows:

1) 10, 50, 100 (improving trend)
2) 10, 100, 50 (no trend established)
3) 50, 10, 100 (no trend established)
4) 50, 100, 10 (no trend established)
5) 100, 50, 10 (deteriorating trend)
6) 100, 10, 50 (no trend established)

If order statistics and distribution plotting techniques could be applied to model each system process (which they can not), the same distribution parameters would be calculated for all six of the above systems. To evaluate one unique repairable system point process, order statistics and distribution plotting and fitting techniques cannot be applied. If, on the other hand, a number of system failure processes are available, order statistics and distribution plotting techniques (as discussed for modeling part TTF data) can be used to evaluate the distribution of time-to-first-failure (TTFF) of the repairable system. This also holds true for any other unique interarrival time (such as time between first and second failure). The appropriate system modeling tools will be presented and discussed in this section.

4.2 Point Process Models

When modeling a single repairable system point process, the two most popular models that have been publicized are the:

- Homogeneous Poisson Process (HPP)
- Nonhomogeneous Poisson Process (NHPP)

The HPP model can be used to describe a process which is stationary and whose time between failures show no trends to increase or decrease as the system ages. This type of repairable system is characterized by a constant rate of occurrence of failure (ROCOF). This constant rate is also called the peril rate, ρ.

The NHPP model can be used to describe a process whose time between failures show trends to increase or decrease as the system ages. The NHPP is a good first approximation for a repairable system because it models a process characterized by a time dependent rate of occurrence of failure or $\rho(t)$.

The procedure for selecting which process model should be applied is provided in Figure 4.2-1. A similar procedure has been recommended by Asher in Reference [55].

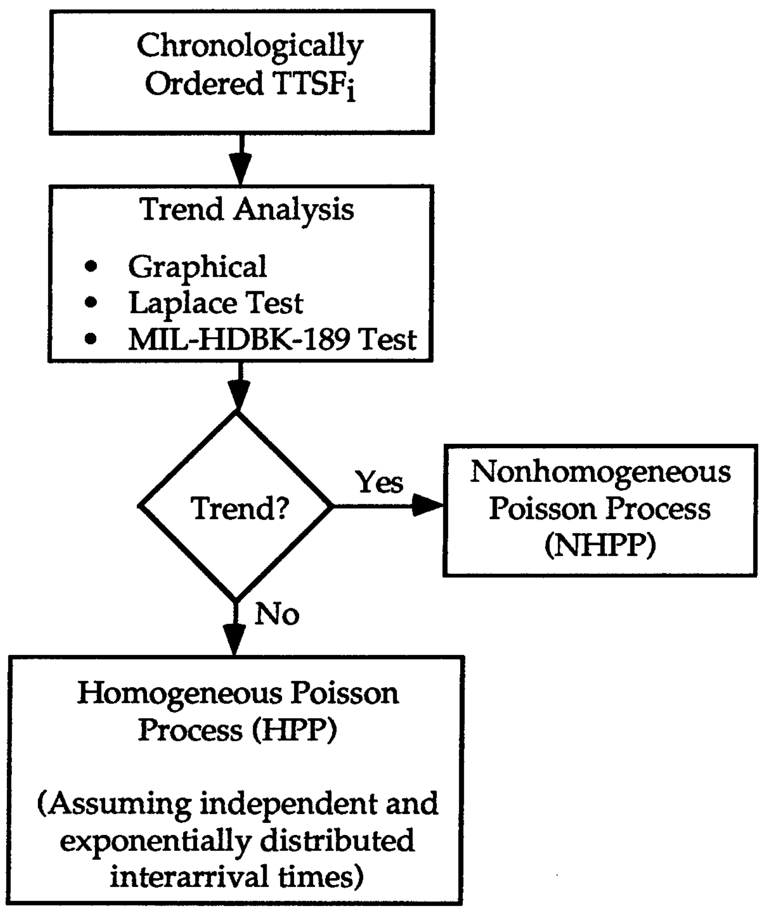


FIGURE 4.2-1: SELECTING THE APPROPRIATE PROCESS MODEL

Both process modeling and trend analysis procedures (HPP, NHPP) will be discussed in Sections 4.2 and 4.3.

4.2.1 Homogeneous Poisson Process (HPP)

The Homogeneous Poisson Process can be used to model a system failure process whose time between failures (TBF_i) are independent and identically exponentially distributed. The interarrival values of the point process (TBF_i) must exhibit no trend to increase or decrease as the system ages. Interarrival values possessing this characteristic are referred to as "random" interarrival values. A system that is neither improving nor deteriorating (i.e., constant rate of occurrence of failure) is generally a good candidate for the HPP model.

The Poisson Process is characterized by the number of failures in any interval from t_1 to t_2 having a Poisson distribution with mean $\rho(t_2 - t_1)$. The Poisson process can be characterized as:

$$P\left\{N(t_2) - N(t_1) = j\right\} = \frac{e^{-\rho(t_2 - t_1)}\left\{\rho(t_2 - t_1)\right\}^j}{j!}, \; j \geq 0 \tag{4-3}$$

where,

N(t) represents the number of failures to time t and ρ is the constant rate of occurrence of failure. Equation (4-3) states the probability of having "j" failures in the interval t_1 to t_2 for a homogeneous Poisson process.

By setting $j = 0$, the probability of no failure in the interval t_1 to t_2 can be determined as:

$$P\left\{N(t_2) - N(t_1) = 0\right\} = e^{-\rho(t_2 - t_1)} \tag{4-4}$$

Equation (4-4) represents the probability of survival, or reliability, in the interval t_1 to t_2 which can be represented as:

$$R(t_1, t_2) = e^{-\rho(t_2 - t_1)} \tag{4-5}$$

4.2.2 Nonhomogeneous Poisson Process (NHPP)

A functional form of time variant rate of occurrence of failure (ROCOF), $\rho(t)$, for the NHPP is:

$$\rho(t) = \lambda\beta t^{\beta-1} \qquad \lambda, \beta > 0, t \geq 0 \tag{4-6}$$

Given a system failure process which contains a trend, the ROCOF or $\rho(t)$, can be determined by maximum likelihood estimators of β and λ. The maximum likelihood estimators of β and λ as shown by Crow[5] are:

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n-1} \ln \frac{TTSF_n}{TTSF_i}} \tag{4-7A}$$

$$\hat{\lambda} = \frac{n}{TTSF_n^{\hat{\beta}}} \tag{4-7B}$$

where,

$TTSF_i$ = Arrival times as identified in Figure 4.1-1
n = Total number of system failure events

As an example, consider the system failure process illustrated in Figure 4.2-2.

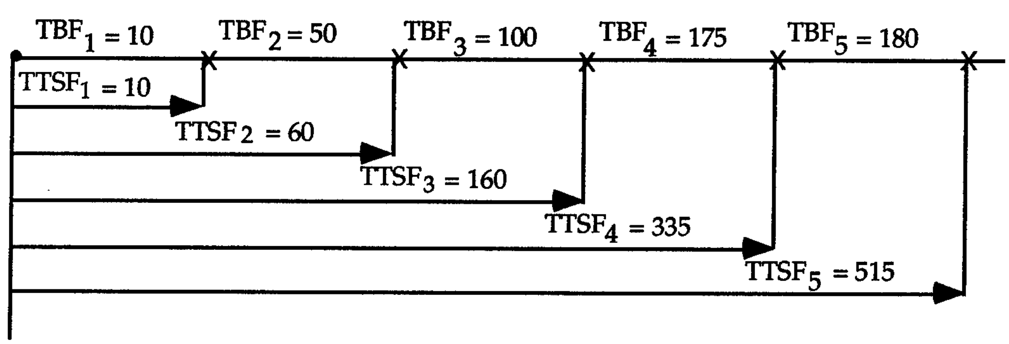


FIGURE 4.2-2: REPAIRABLE SYSTEM FAILURE PROCESS, EXAMPLE

5 Crow, L.H., "Reliability Analysis for Complex Repairable Systems," Reliability and Biometry, F. Proschan and R.J. Serfling, eds., SIAM, Philadelphia, pp. 379-410, 1974.

Using Equation (4-7A), calculate β:

$$\hat{\beta} = \frac{5}{\ln\frac{515}{10} + \ln\frac{515}{60} + \ln\frac{515}{160} + \ln\frac{515}{335}} = .65$$

Using Equation (4-7B), calculate $\hat{\lambda}$:

$$\hat{\lambda} = \frac{5}{515^{.65}} = \frac{5}{57.9} = .086$$

Substitution of $\hat{\beta}$ and $\hat{\lambda}$ into Equation (4-6) yields:

$$\rho(t) = (.086)(.65)t^{.65 - 1}$$

$$\rho(t) = .056\, t^{-.35}$$

The expected number of failures in the interval zero to t, V(t), is given by the following:

$$V(t) = \int \rho(t)dt \tag{4-8A}$$

Substituting Equation (4-6) into Equation (4-8A) yields:

$$V(t) = \lambda t^{\beta} \tag{4-8B}$$

Using our example, the expected number of failures after 300 hours is:

$$V(300) = (.086)300^{.65} = 3.5$$

This value of 3.5 failures corresponds with the expected number of failures after 300 hours for the system shown in Figure 4.2-2.

4.3 Trend Analysis of System Failure Data

In this section we will present two procedures for evaluating if a trend exists in a system failure process. The two procedures for evaluating trends are:

a) Graphical Plot of cumulative failures versus Cumulative Operating Time Using Linear Scales

b) Laplace test statistic

Each of these trend analysis procedures is easy to apply and interpret.

As indicated in Table 4.3-1, the determination whether or not a trend (i.e., increasing or decreasing TBF_i) exists is essential in selecting the appropriate model for the process.

4.3.1 Plotting Cumulative Failures vs. Cumulative Operating Time

Let us now consider the two system failure processes defined in Table 4.3-1.

TABLE 4.3-1: SYSTEM FAILURE PROCESS DATA

Failure Order Number (i)	System A Arrival Times ($TTSF_i$)	System B Arrival Times ($TTSF_i$)
1	15	177
2	42	242
3	74	293
4	117	336
5	168	368
6	233	395
7	410	410

The data for system A was intentionally fabricated to represent an increasing trend in the time between failures (TBF_i) which are:

A: 15, 27, 32, 43, 51, 65, 177

The data for system B was intentionally fabricated to represent a decreasing trend in time between failures (TBF_i) which are:

B: 177, 65, 51, 43, 32, 27, 15

Both of these systems (A, B) can be evaluated by constructing a plot of cumulative failures versus cumulative test time on linear scales as shown in Figure 4.3-1. The data from Table 4.3-1 was used to generate each curve.

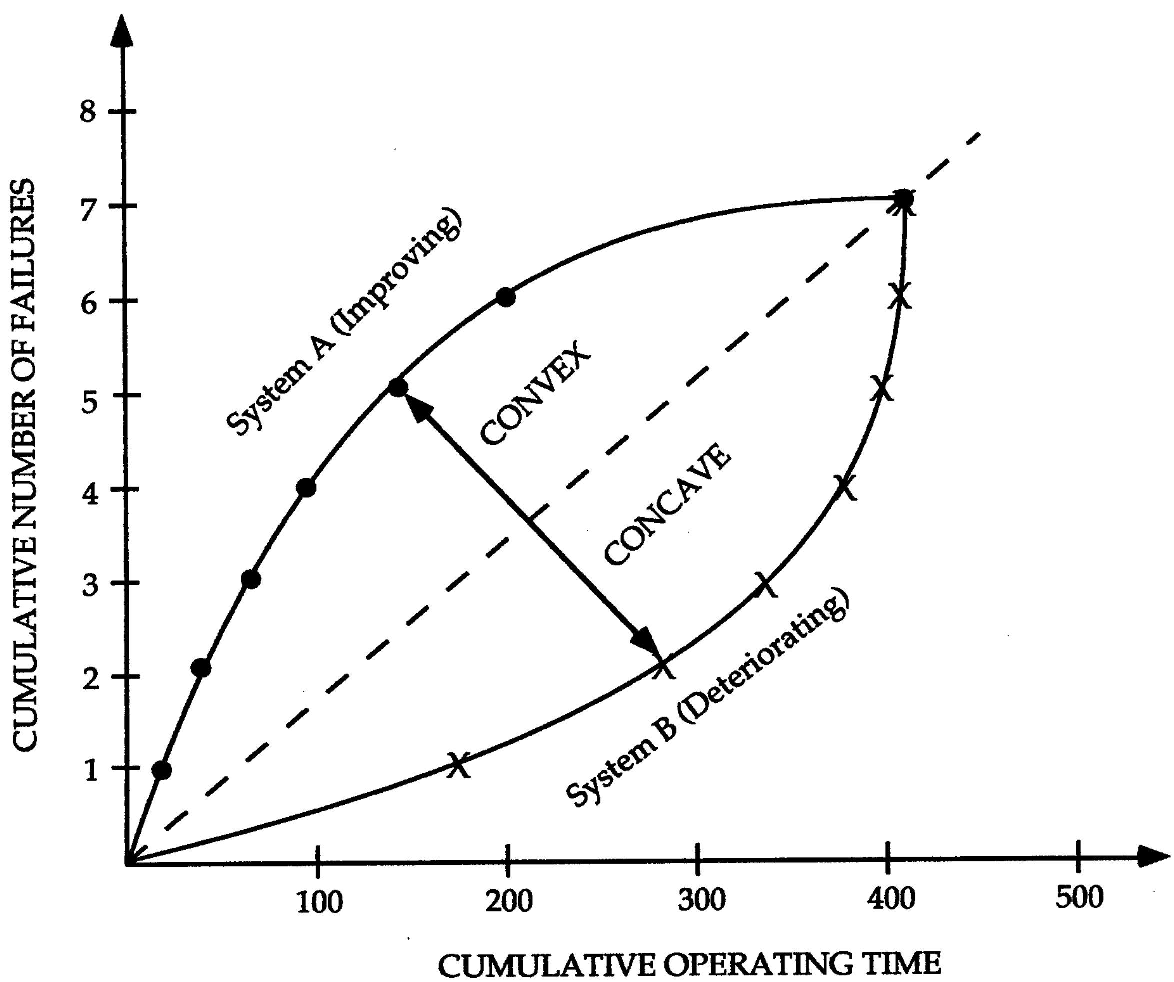


FIGURE 4.3-1: CUMULATIVE FAILURES VS. CUMULATIVE OPERATING TIME

Using Figure 4.3-1 as a visual reference, we can conclude that failure processes (such as System A) which exhibit a convex curve on a plot of cumulative failures versus cumulative operating time using linear scales represent improving systems (i.e., TBF_i are tending to increase). Failure processes (such as System B) which exhibit a concave curve on a plot of cumulative failures versus cumulative operating time represent deteriorating systems (i.e., TBF_i are tending to decrease).

This graphical technique provides a simple but effective means to visually assess whether or not a trend exists in a system failure process and can be applied prior to modeling using the HPP or NHPP.

4.3.2 Laplace Test Statistic

Pierce Simon Laplace (1749 - 1827) was one of the great mathematicians of the eighteenth century and was responsible for many of the statistical theorems which are still in use today - one being the central limit theorem and another being the much less known Laplace statistic. Here we adopt his principle to evaluate whether or not a trend is present for failure events of a system.

As with the graphical method discussed in Section 4.3.1, the Laplace test statistic can also be used to determine if sequential interarrival times (TBF_i) are tending to increase, decrease or remain the same. The Laplace test statistic for a process with "n" failures is:

$$U = \frac{\left[\left(\sum_{i=1}^{n-1} TTSF_i\right) / (n-1)\right] - (TTSF_n / 2)}{TTSF_n \sqrt{1/(12(n-1))}} \qquad (4\text{-}9)$$

The conclusions which can be rendered based the Laplace statistic, U, are:

a) U approximately equal to zero indicates the lack of trend

b) U greater than zero indicates interarrival values (TBF_i) are tending to decrease (i.e., system deterioration)

c) U less than zero indicates interarrival values (TBF_i) are tending to increase (i.e., system improvement)

If we again utilize the system failure process definitions of Figure 4.3-1 and calculate the Laplace statistic for System A then B:

System A:

Given: $n = 7$

$TTSF_n = 410$

$$\sum_{i=1}^{n-1} TTSF_i = 646$$

Calculate the Laplace statistic using Equation (4-9):

$$U = \frac{(646/6) - (410/2)}{410\sqrt{1/72}}$$

$U = -2.01$ (System TBF_i tends to increase)

System B:

Given: $n = 7$

$TTSF_n = 410$

$$\sum_{i=1}^{n-1} TTSF_i = 1811$$

Calculate the Laplace statistic using Equation (4-9):

$$U = \frac{(1811/6) - (410/2)}{410\sqrt{1/72}}$$

$U = +2.00$ (System TBF_i tends to decrease)

5.0 MONTE CARLO SIMULATION

The term "simulation" refers to a numerical technique for conducting experiments on a digital computer which involves evaluating certain types of mathematical and logical models that describe system behavior. One particular type of simulation is stochastic simulation, which involves experimenting with a model over time and includes sampling random variables from probability distributions. Stochastic simulation is often termed the Monte Carlo method and uses pseudo-random numbers for the solution of a model. The pseudo-random numbers are generated by recursive algorithms. Simulation models are useful where closed-form mathematical solutions are impossible or very time-consuming.

The Monte Carlo method provides a simulation technique for predicting system performance information from part reliability characteristics. This method has been applied in a variety of ways to predict mechanical system reliability. Simulation offers a numerical approach for evaluating system reliability where few restrictions are placed on the complexity of the model(s).

The Monte Carlo method consists of repeated numerical random sampling of a given model. The process is essentially "synthetic experimentation" where many systems are built by computer calculations and the system performances are evaluated and summarized to obtain system reliability. A flow chart illustrating a typical process of Monte Carlo simulation is shown in Figure 5.0-1.

Two cases involving the application of Monte Carlo simulation for determining mechanical system reliability are illustrated. For Case One (1), the system reliability block diagram and the part reliabilities are known. For Case Two (2), the system reliability block diagram is known along with the stress - strength distributions for each part.

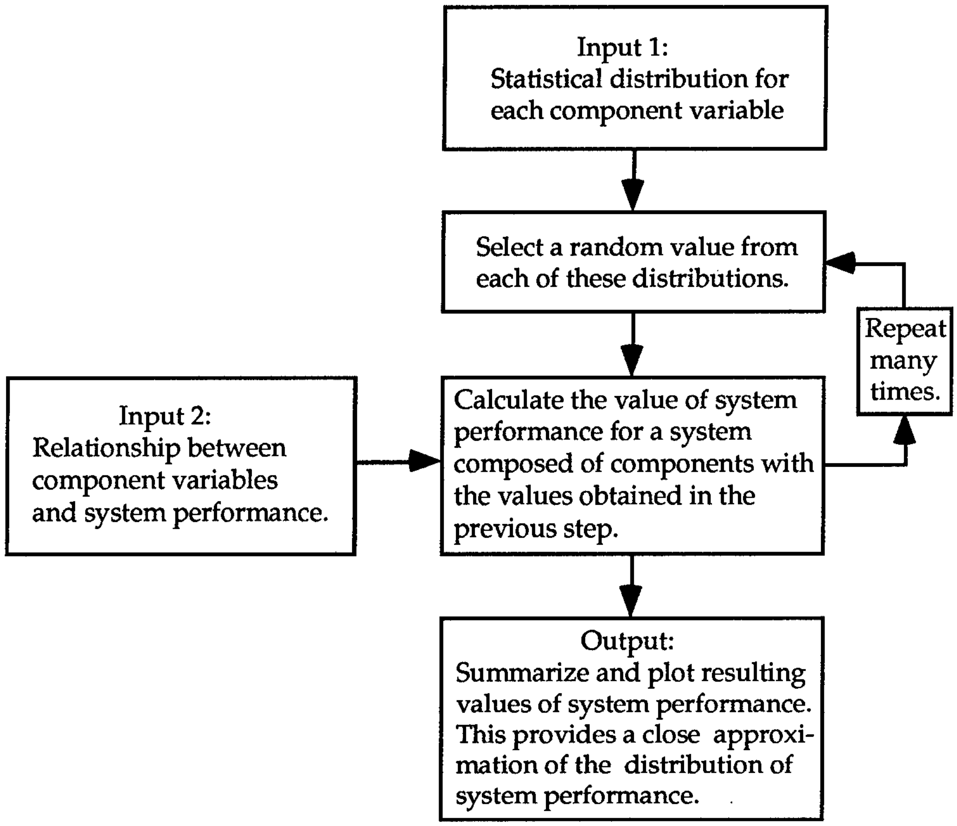


FIGURE 5.0-1: FLOW CHART OF MONTE CARLO SIMULATION METHOD [10]

Case 1

The following procedure can be applied to determine system reliability using the Monte Carlo method when the system reliability block diagram and part reliabilities are known. It is assumed that the reliabilities are determined from an identical constant value of cumulative operating time. Other times can be considered by iterating Steps 1-5.

Step 1: Given N actual part reliabilities R_i^A (i = 1, 2, ..., N) and the system reliability block diagram.

Step 2: Generate N independent uniform variates (random numbers) in the interval 0, 1 and designate them the required part reliabilities R_i^R (i = 1, 2, ..., N).

Step 3: Compare the required part reliabilities to the actual part reliabilities and consider the i^{th} part a failure when $R_i^R \geq R_i^A$ $(i = 1, 2, 3, ..., N)$

Step 4: Determine system success or failure using the system reliability block diagram and the information from Step 3.

Step 5: Steps 2 to 4 are repeated many times and a system failure or success is recorded each time. The system reliability is then estimated as:

$$R_{sys} \cong \frac{\text{Number of system successes}}{\text{Total number of Monte Carlo trials}} \tag{5-1}$$

As an example, the procedure just described in Steps 1- 5 is applied to a mechanical system whose reliability block diagram is shown in Figure 5.0-2. A simple system was used as an illustration of the procedure. The practical application of the Monte Carlo method involves more complex systems and the use of high-speed digital computers to generate many simulated trails.

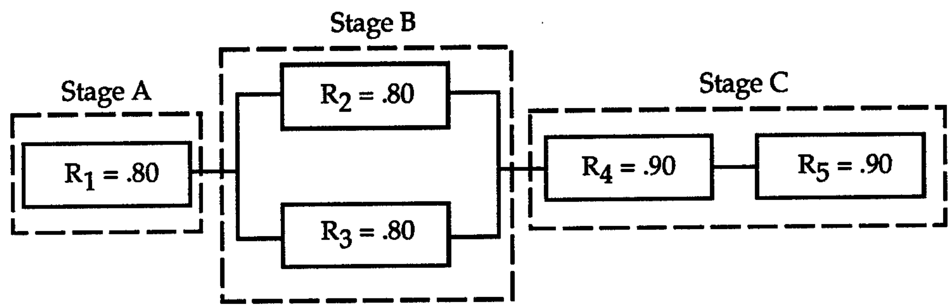


FIGURE 5.0-2: SYSTEM RELIABILITY BLOCK DIAGRAM

Seven iterations of Steps 2 - 4 were performed to illustrate this method, and the results are presented in Table 5.0-1. In actual application, many trials (i.e., greater than 1,000) would be simulated. A table of random values from the uniform distribution was used to generate part reliabilities. The system reliability calculated from the results in Table 5.0-1, Column 9 and Equation (5-1) yields:

$$R_{sys} \cong \frac{\text{Number of system successes}}{\text{Total number of Monte Carlo trials}}$$

$$R_{sys} \cong 5/7$$

$R_{sys} \cong .71$ (From only seven Monte Carlo trials)

TABLE 5.0-1: SEVEN MONTE CARLO TRIALS TO ESTIMATE THE SYSTEM RELIABILITY OF FAILURE 5.0-2

1 Random Value Generated For R_1	2 Result For Stage A	3 Random Value Generated For Block R_2	4 R_3	5 Result For Stage B	6 Random Value Generated For Block R_4	7 R_5	8 Result For Stage C	9 System Result*
.01	Success	.28	.51	Success	.39	.10	Success	Success
.38	Success	.37	.63	Success	.90	.49	Failure	Failure
.08	Success	.48	.35	Success	.25	.23	Success	Success
.99	Failure	.02	.50	Success	.75	.03	Success	Failure
.13	Success	.34	.44	Success	.05	.25	Success	Success
.66	Success	.48	.07	Success	.52	.17	Success	Success
.31	Success	.80	.60	Success	.56	.13	Success	Success

*System Success = Successful Stage A and Stage B and Stage C

Case 2

The following procedure can be applied to determine system reliability using Monte Carlo simulation when the system reliability block diagram and the statistical distributions of stress-strength for each part are known.

Step 1: Given N parts in the reliability block diagram and the stress (s_i) and Strength (S_i) distributions for each part (i = 1, 2, 3, ..., N).

Step 2: For s_i and S_i (i = 1, 2, 3, ..., N) determine the proper equation for calculating the random stresses $\left(x'_{s_i}\right)$ and random strengths $\left(x'_{S_i}\right)$ from Table 5.0-2.

Step 3: Calculate x'_{s_i} and x'_{S_i} (i = 1, 2, 3, ..., N) which requires the generation of random numbers from either the random standard normal distribution (R_N) or the random standard uniform distribution (R_U), depending on which is required.

Step 4: Compare the random stresses $\left(x'_{s_i}\right)$ and random strengths $\left(x'_{S_i}\right)$ and consider the i^{th} part a failure when:

$$x'_{s_i} \geq x'_{S_i} \quad (i = 1, 2, 3, \ldots, N) \text{ or stress greater than strength}$$

TABLE 5.0-2: GENERATION OF RANDOM VALUES FROM VARIOUS DISTRIBUTIONS GIVEN RANDOM STANDARD NORMAL (R_N) AND RANDOM STANDARD UNIFORM (R_U) VALUES*

Distribution to be Simulated	Probability Density Function	Procedure to Obtain Random Value x'
Exponential	$f(x) = \lambda e^{-\lambda t}$	$x' = -\frac{1}{\lambda} \ln R_U$
Weibull	$f(x) = \frac{\beta}{\alpha^\beta} x^{\beta-1} \exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right]$	$x' = \alpha\left(-\ln R_U\right)^{1/\beta}$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$	$x' = \mu + \sigma R_N$
Log-Normal	$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[\frac{(\ln x-\mu)^2}{2\sigma^2}\right]$	$x' = e^{\mu + \sigma R_N}$
Gamma	$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}$	$x' = -\beta \ln \prod_{i=1}^{\alpha} R_{U_i}$

Note: * R_N is random value from normal distribution with $\mu = 0$, $\sigma = 1$. R_U is random value from uniform distribution over interval (0, 1). When more than one value is required, a typical value is designed as R_{N_i} or R_{U_i}. All values are taken independently of one another.

Step 5: Determine system success or failure using the system reliability block diagram and information from Step 4

Step 6: Steps 3 to 5 are repeated many times and a system failure or success is recorded each time. The system reliability is then estimated as:

$$R_{sys} \cong \frac{\text{Number of system successes}}{\text{Total number of Monte Carlo trials}}$$

As the complexity of a system increases and analytical approaches become too consuming in terms of manhours, simulation procedures structured for computer application become more advantageous to predict system reliability. Recent advances in simulation methodologies and available software have made simulation one of the most widely used and accepted tools in system analysis.

6.0 FAILURE MODE EVALUATION TECHNIQUES FOR SYSTEMS

Two systematic methods for evaluating the consequence of failure within a system are presented in this section. They are:

- Failure Mode, Effects and Criticality Analysis (FMECA)
- Fault Tree Analysis (FTA)

Each of these evaluation tools has been well documented in numerous references such as: Fault Tree Analysis Application Guide (Reference [73]), Fault Tree Handbook (Reference [85]) and MIL-STD-1629A (Reference [78]). The intent of the material contained in this section is to provide an overview of the procedures required to perform each analysis and relate their use to mechanical applications.

The approach utilized in the FMECA is different from that taken by the FTA. The FMECA is a "bottom up" approach while the FTA is a "top down" approach. The reason for each of these descriptions is illustrated in Figure 6.0-1.

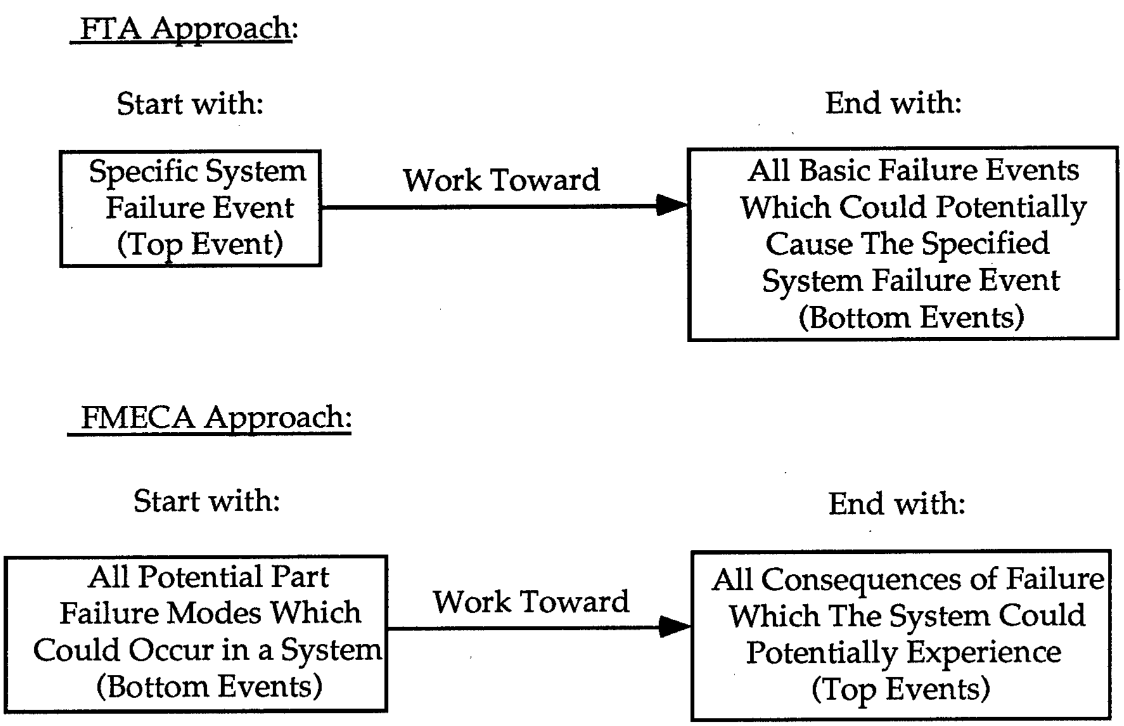


FIGURE 6.0-1: COMPARING THE APPROACH OF FMECA AND FTA

The approach/procedures for performing each of these analyses will be discussed in further detail in Section 6.1 and 6.2.

Table 6.0-1 is provided to assist the analyst in determining which analysis, FMECA or FTA, is preferred based on the individual circumstances. These recommendations are made based on the abilities of each approach to best satisfy the stated circumstance or requirement. For example, the FTA has the ability to handle failures other than hardware failures, therefore, FTA would be preferred over FMECA if nonhardware failures require consideration.

TABLE 6.0-1: FTA VS. FMECA SELECTION CRITERIA

Circumstance	Fault Tree Preferred	FMECA Preferred
Safety of the general public or operating and maintenance personnel is the primary concern	X	
A small number of clearly differentiated 'Top Events' are explicitly defined	X	
Inability to clearly define a small number of 'Top Events'		X
Completion of the Entire Mission is of critical importance	X	
Any number of potentially successful missions are possible		X
All possible failure modes are of concern		X
There is a high potential for "Human Error" contributions	X	
There is a high potential for "Software Error" contributions	X	
A numerical "Risk Evaluation" is the primary concern	X	
The system architecture is highly complex and/or it contains highly interconnected functional paths	X	
The system is basically of a linear architecture with little human or software intervention		X
The system is not repairable once the mission commences	X	
Require lowest cost analysis	X	
Require the most timely analysis	X	
Require reliability improvement support		X

6.1 Failure Mode, Effects and Criticality Analysis (FMECA)

The Failure Mode, Effects and Criticality Analysis (FMECA) is a systematic design evaluation procedure whose purpose is to identify potential failure modes and to assess their effects throughout the system and to define failure mode criticality which provides quantitative assessment of each failure mode based on frequency and consequence. FMECA has become one of the most effective system design analysis techniques used in reliability engineering. A properly performed FMECA is used to support a wide range of engineering activities such as design and maintenance.

The FMECA provides:

1) The design engineer with a method of selecting a design with a high probability of operational success.

2) Design engineering with a uniform method for assessing failure modes and their effects on operational success of the system.

3) Early visibility of system problems.

4) A list of possible failures which can be ranked according to their category of effects and probability of occurrence.

5) Identification of single failure points critical to success.

6) Early criteria for test planning.

7) A basis for design and location of performance monitoring and fault sensing devices and other built-in automatic test equipment.

8) A tool which serves as an aid in the evaluation of proposed design, operational, or procedural changes and their impact on success.

FMECA utilizes inductive logic in a "bottom up" approach. Beginning at the lowest level of the system hierarchy (e.g., part) and from a knowledge of the failure modes of each part, the analyst traces up through the system hierarchy to determine the effect that each failure mode will have on system performance. This differs from fault tree analysis which utilizes deductive logic in a "top down" approach. In fault tree analysis, the analyst assumes a system failure and traces down through the system hierarchy to determine the event or series of events that could cause such a failure.

Analysts performing an FMECA must first acquire a full understanding of the design and its operation. It is important to understand the function of each part and how it interfaces with other parts. Toward this end, any information available on the design must be obtained. A typical list of preferred documentation may include: specifications, drawings, stress analyses, test results, reliability predictions, bill of materials, theory of operations, operating manuals, etc. Once an understanding of how a system works is acquired, then the focus can shift to determine how it can fail.

The FMECA is a combination of two analysis procedures which are:

1) Failure Mode and Effects Analysis (FMEA)
2) Criticality Analysis (CA)

The FMEA is an analysis procedure which identifies each potential failure mode in a system. Each failure mode is then assessed in terms of its effects at the local, next higher assembly and system levels. Finally, each failure mode is given a severity classification according to the system level effects. The initial FMEA should be done early in the conceptual phase when design criteria, mission requirements, and preliminary designs are being developed to evaluate the design approach and to compare the benefits of competing design configurations.

The FMEA will provide quick visibility of the most obvious failure modes and identify potential single failure points, some of which can be eliminated with minimal design modifications. As the design definitions become more refined, the FMEA can be expanded to successively more detailed levels. When changes are made in system design to remove or reduce the impact of critical failure modes, the FMEA should be updated to ensure that all predictable failure modes in the new design are considered.

The analysis approach to be used for the FMEA will generally be dictated by variations in design complexity and the available data. There are two primary approaches to accomplish an FMEA.

- Functional FMEA Approach - The functional approach is normally used when hardware items cannot be uniquely identified. Each identified failure mode is assigned a severity classification which can be utilized during design iterations to establish priorities for corrective actions. The functional FMEA should commence after the design process has delivered a functional block diagram of the system but has not yet identified a specific hardware implementation. It is the first FMEA to be performed and should be updated throughout the design iteration process or as corrective actions are implemented.

- Hardware FMEA Approach - The hardware approach is normally used when hardware items can be uniquely identified from schematics, drawings, and other engineering and design data. The hardware approach is normally utilized in a part level up fashion. Each identified failure mode is assigned a severity classification which will be utilized during design to establish priorities for corrective actions. The hardware FMEA should commence after the design process has delivered a schematic diagram with all system items or parts defined.

For complex systems, a combination of the functional and hardware approaches may be considered. The FMEA may be performed as a hardware analysis, a functional analysis, or a combination analysis and is ideally initiated at the part, circuit or functional level and proceeds through increasing indenture levels until the FMEA for the system is complete.

An optimum set of information has been assembled for performing the FMEA. This information is typically arranged in the format shown in Figure 6.1-1. Figure 6.1-1 represents a typical FMECA worksheet.

FAILURE MODE AND EFFECTS ANALYSIS

SYSTEM ____	DATE ____
INDENTURE LEVEL ____	SHEET ____ OF ____
REFERENCE DRAWING ____	COMPILED BY ____
MISSION ____	APPROVED BY ____

					Failure Effects						
ID Number	Item/Functional Identification (Nomenclature)	Function	Failure Modes and Causes	Mission Phase/ Operational Mode	Local Effects	Next Higher Level	End Effects	Failure Detection Method	Compen-sating Provisions	Severity Class	Remarks

FIGURE 6.1-1: FMEA WORKSHEET FORMAT

Each of the significant data elements shown in Figure 6.1-1 is defined as follows:

- Identification number. A serial number or other reference designation is assigned for traceability purposes.

- Item/functional identification. The name or nomenclature of the item or system function, being analyzed for failure mode and effects, is listed.

- Function. A concise statement of the function performed by the hardware item are listed. This includes both the inherent function of the part and its relationship to interfacing items.

- Failure modes and causes. All predictable failure modes for each item analyzed shall be identified and described. Potential failure modes are determined by examination of item outputs and functional outputs identified in applicable block diagrams and schematics. Failure modes of the individual item function are postulated on the basis of the stated requirements in the system definition narrative and the failure definitions included in the ground rules. The most probable causes associated with the postulated failure mode are identified and described. Since a failure mode may have more than one cause, all probable independent causes for each failure mode are identified and described.

- Mission phase/operational mode. A concise statement of the mission phase and operational mode in which the failure occurs. Where subphase, event, or time can be defined from the system definition and mission profiles, the most definitive timing information should also be entered for the assumed time of failure occurrence.

- Local or Primary Effects. Local effects concentrate specifically on the impact an assumed failure mode has on the operation and function of the item in the indenture level under consideration. The consequences of each postulated failure affecting the item shall be described along with any second-order effects which result. The purpose of defining local effects is to provide a basis for evaluating compensating provisions and for recommending corrective actions. It is possible for the "local" effect to be the failure mode itself.

- Next Higher Level or Secondary Effects. Next higher level effects concentrate on the impact an assumed failure has on the operation and function of the items in the next higher indenture level above the indenture level under consideration. The consequences of each postulated failure affecting the next higher indenture level shall be described. If analyzing a seal in a pump, the effect that the failed seal has on the pumps function would be described at this level.

- <u>End or System Effects</u>. End effects evaluate and define the total effect an assumed failure has on the operation, function, or status of the uppermost system. The end effect described may be the result of a multiple failure. For example, failure of a safety device may result in a catastrophic end effect only in the event that both the prime function goes beyond the limit for which the safety device is set and the safety device fails.

- <u>Failure Detection Method</u>. Describe the methods by which occurrence of a failure mode is detected by the operator or maintenance personnel. The failure detection means, such as visual or audible warning devices, automatic sensing devices, sensing instrumentation, other unique indications, or none, should also be identified here.

- <u>Failure Compensation Method</u>. Identify corrective design or other actions required to eliminate the failure or control the risk. This step is required to record the true behavior of the item in the presence of a failure. The analyst should describe design compensating provisions that will: (1) nullify the effects of a failure, (2) control or deactivate system items to halt generation or propagation of failure effects, or (3) activate backup or standby items or systems. Design compensating provisions can include redundant items that allow continued and safe operation, safety or relief devices such as monitoring or alarm provisions which permit effective operation or limit damage and alternative modes of operation such as backup or standby items or systems.

- <u>Severity Classification</u> - Each failure mode should be evaluated in terms of the worst potential consequences which may result. A code will be assigned describing the worst possible incidence of this failure. This code is the severity classification code. Severity classifications are assigned to provide a qualitative measure of the worst potential consequences resulting from design error or item failure. A severity classification is assigned to each identified failure mode and each item analyzed. Severity classification categories which are consistent with various military standards are defined as follows:

  - **Category I - Catastrophic:** A failure which may cause death or system loss.

  - **Category II - Critical:** A failure which may cause severe injury, major property damage, or major system damage which will result in mission loss.

  - **Category III - Marginal:** A failure which may cause minor injury, minor property damage, or minor system damage which will result in delay or loss of availability or mission degradation.

- **Category IV - Minor**: A failure not serious enough to cause injury, property damage, or system damage, but which will result in unscheduled maintenance or repair.

Where it may not be possible to identify an item or a failure mode according to the loss statements in the four categories above, similar loss statements based upon loss of system inputs or outputs can be developed.

Many experienced analysts may choose to customize the severity classifications and derive a set of classifications which are more explanatory for a particular system. For example, the following severity classifications were used for a missile fuze FMEA:

1) No effect on mission
2) Fuze does not detonate after launch
3) Fuze detonates too high after electrical arming
4) Fuze detonates too low after electrical arming
5) Fuze detonates after launch but before electrical arming
6) Fuze detonates before launch

Next we will consider the criticality analysis (CA). The CA is an analysis procedure for associating criticality numerics with each failure mode. The CA complements the FMEA and is dependent upon information developed in that analysis. The CA is typically performed in conjunction with or following the FMEA. The CA is a valuable tool for maintenance and logistic support since failure modes which have a high probability of occurrence (high criticality numbers) and a significant consequence can be identified and assessed in terms of potential impact on the requirements for the system.

The analysis approach to be used for the CA will generally be dictated by the availability of failure rate data. There are two approaches for accomplishing the CA. One is the qualitative approach which is appropriate only when failure rate data is not available. The preferred method is the quantitative approach which is utilized when failure rate data is available.

A criticality analysis worksheet is shown in Figure 6.1-2. The criticality analysis complements the FMEA with additional data elements which are indicated by asterisks in Figure 6.1-2.

The following describe the data elements necessary to perform either a qualitative or quantitative criticality analysis:

- Failure Probability/Failure Rate Data Source. When a qualitative CA is performed, failure modes are assessed in terms of probability of occurrence, and the failure probability of occurrence level must be shown in this column. When failure rate data are available, a quantitative CA can be performed and criticality numbers may be calculated. In this case, the data source of the failure rates used in each calculation shall be listed in this column. When a failure probability is listed, the remaining columns are not required.

- Failure Effect Probability (β). The β value is the conditional probability that the failure effect will result in the identified criticality classification, given that the failure mode occurs. The β value represents the analyst's judgment as to the conditional probability the loss will occur and should be quantified in general accordance with the values in Table 6.1-1.

TABLE 6.1-1: TYPICAL FAILURE EFFECT PROBABILITIES (β)

FAILURE EFFECT	β VALUE
Actual Loss	1.00
Probable Loss	> 0.10 to < 1.00
Possible Loss	> 0 to 0.10
No Effect	0

- Failure mode ratio (α). The fraction of the part failure rate (λ_p) related to the particular failure mode under consideration shall be evaluated by the analyst and recorded here. The failure mode ratio is the probability expressed as a decimal fraction that the part or item will fail in the identified mode. RAC publication "Failure Mode/Mechanism Distributions" (FMD-91) provides generic failure mode ratio (α) data such as shown in Table 6.1-2.

- Part failure rate (λp). The part failure rate (λp) from the appropriate reliability prediction or failure rate data source such as RAC publication "Nonelectronic Parts Reliability Data" (NPRD-91) shall be listed.

CRITICALITY ANALYSIS

SYSTEM __________
INDENTURE LEVEL __________
REFERENCE DRAWING __________
MISSION __________

DATE __________
SHEET _____ OF _____
COMPILED BY __________
APPROVED BY __________

ID Number	Item/Functional Identification (Nomenclature)	Function	Failure Modes and Causes	Mission Phase/ Operational Mode	Severity Class	Failure Probability Failure Rate Data Source	Failure Effect Probability (β)	Failure Mode Ratio (α)	Failure Rate (λ_p)	OP Time (t)	Failure Mode Crit # $C_m = \beta \alpha \lambda_p t$	Item Crit # $C_r = \Sigma (C_m)$	Remark

FIGURE 6.1-2: CRITICALITY ANALYSIS WORKSHEET

TABLE 6.1-2: EXAMPLE FAILURE MODE RATIO (α) DATA

Device Type	Failure Mode	Failure Mode Probability (α)
Accumulator	Leaking Seized Worn Contaminated	.47 .23 .20 .10
Actuator	Spurious Position Change Binding Leaking Seized	.36 .27 .22 .15
Adapter	Physical Damage Out of Adjustment Leaking	.33 .33 .33
Alarm	False Indication Failure to Operate on Demand Spurious Operation Degraded Alarm	.48 .29 .18 .05
Antenna	No Transmission Signal Leakage Spurious Transmission	.54 .21 .25
Battery, Lithium	Degraded Output Startup Delay Short Open	.78 .14 .06 .02
Battery, Lead Acid	Degraded Output Short Intermittent Output	.70 .20 .10
Battery, Rechargeable, Ni-Cd	Degraded Output No Output	.72 .28
Bearing	Binding/Sticking Excessive Play Contaminated	.50 .43 .07
Belt	Excessive Wear Broken	.75 .25

- Operating time (t). The operating time in hours or the number of operating cycles of the item per mission is derived from the system definition and listed on the worksheet.

- Failure mode criticality number (C_m). The value of the failure mode criticality number (C_m) is calculated and listed on the worksheet. (C_m) is the portion of an item's criticality number due to the single failure mode under investigation for a particular severity classification. For each particular failure mode severity classification, the (C_m) is calculated with the following formula:

$$C_m = \beta\alpha\lambda_p t \tag{6-1}$$

where,

C_m = Criticality number for failure mode
β = Conditional probability of mission loss
α = Failure mode ratio
λ_p = Part failure rate
t = Duration of applicable mission phase usually expressed in hours or number of operating cycles

- Item criticality numbers (C_r) - The second criticality number calculation is for the item under analysis. Item criticality numbers (C_r) for each system item under investigation is calculated and listed on the worksheet. An item may be considered a component, assembly or function depending on the detail of analysis or level of indenture which the FMECA is being performed. For a particular severity classification and mission phase, the (C_r) for an item is the sum of the failure mode criticality numbers (C_m), under the severity classification and may also be calculated using the following formula:

$$C_r = \sum_{n=1}^{j} \left(\beta\alpha\lambda_p t\right)_n \qquad n = 1, 2, 3, \ldots j \qquad \text{or} \quad C_r = \sum_{n=1}^{j} \left(C_m\right)_n \tag{6-2}$$

where,

C_r = Criticality number for the item.

n = The failure modes in the items that fall under a particular severity classification.

j = Total number of failure modes in the item under the severity classification.

Other customized variations of the FMECA worksheet have been developed to serve specific individual requirements. Such an example is shown in Figure 6.1-3. This FMECA worksheet contains eleven fields (columns) of data containing

Failure Mode, Effects and Criticality Analysis

Sheet ____ of ____

Indenture Level No. 1: ____________

Indenture Level No. 2: ____________

Indenture Level No. 3: ____________

Indenture Level No. 4: ____________

Drawing: ____________

Contract or Project No.: ____________

Organization: ____________

Prepared by: ____________

Approved by: ____________

Date: ____________

Part Characteristics				Failure Mode		Effects of Failure		Severity Classification	Modal Failure Rate	Comments
Name	I.D. Number	Function	Failure Rate	Description	Probability	Local Effect	End Effect			

FIGURE 6.1-3: CUSTOMIZED FMECA WORKSHEET

information on each part considered. These fields are described as follows:

- Name

  A label which designates a particular part.

- I.D. Number

  An alphanumeric code used to designate the part.

- Function

  A description of the part's major role.

- Part Failure Rate (λ_p)

  A numerical value having units of failures per unit time. Normally, the failure rate is obtained from a part reliability prediction.

- Failure Mode Description

  Identifies the mode in which the part could fail (refer to Table 3.1-1).

- Failure Mode Probability

  A percentage which indicates the probability of occurrence of each individual failure mode. Same as the failure mode ratio.

- Local Effect

  A description of the immediate effect of failure by the respective failure mode.

- End Effect

  A description of the end effect (created by the respective failure mode) resulting from the propagation of failures through the system.

- Failure Classification

  A code that identifies a level of criticality for each individual failure mode. Normally, the failure classification is relative to the system performance. Equivalent to the severity classification.

- Modal Failure Rate

  A numerical value that is the product of the part failure rate and the failure mode probability. The modal failure rate along with the failure classification are used to assess the criticality of each individual failure mode.

- Comments

  Any other relevant items are listed in this column. Items such as references, duty cycle, Weibull shape parameter and lubricant are examples.

The information placed in the eleven columns of Figure 6.1-3 may include, for example:

Part Characteristics

Name:	Oil Seal (Radial Lip)
I.D. Number:	643293
Function:	Prevents the flow of lubricant at the interface between the cutting cylinder shaft and frame.
Failure Rate:	.80 (failures/10^6 part hours)

Failure Mode

Description:	Leakage
Probability of Occurrence:	.75

Effects of Failure

Local Effect:	Lubricant is lost from tapered roller bearing cavity.
End Effect:	Abrasives collect at lip seal and shaft interface and accelerated wear of both.

Failure Classification: Degraded System Operation (DSO)

Modal Failure Rate: .60 (failures/10^6 part hours)

Comments: Failure rate based on a shaft speed of 1725 rpm.

The "automated FMECA" offers significant improvement over the manual FMECA. An automated FMECA can provide the necessary traceability from design elements to the failure effects as well as from failure effects to the design elements. This dual line of traceability was not readily obtainable in the original "manual tabular" method. The automated technique should utilize a clear and straightforward approach to cross-referencing between design elements (e.g., parts) and failure effects. This allows the practical use of the FMECA by those involved in various disciplines (e.g., reliability, maintenance, design, management, etc.) relating to product development.

When performing an automated FMECA, the database and report generator should be structured so that (at least) two major listings are generated as output, namely the "design element" listing and the "effects" listing. An example of each listing is shown in Figure 6.1-4 and Figure 6.1-5. Lind (Reference [71]) described each of these listings as follows:

The Design Element Listing

> "The design element listing identifies the design elements and their failure modes in the order that the analysis was coded. Comments can also be included in this listing. To allow space for additional FMEA data, the effect descriptions are not printed but instead are referenced by effect number. Listing by design elements allows verification that all failure modes of all design elements were considered in the analysis. Failure mode causes preventive measures can be included in this listing."

The Effect Listing

> "The effect listing identifies the effects with the design element and applicable failure modes that could cause those effects. The effect listing provides a convenient method of identifying all pertinent information such as design elements, failure modes, criticality, and failure rates that should be considered in the evaluation of the effects."

It is interesting to note, that the information contained in the effect listing shown in Figure 6.1-5 would provide an excellent troubleshooting document for a system. All that is required is to adopt a standard set of end effects at the system level. Many FMECAs performed do not take advantage of this highly useful feature of the FMECA.

6.2 Fault Tree Analysis (FTA)

Fault Tree Analysis (FTA) is a failure mode evaluation technique that can be applied to any system. This technique (1) identifies undesired event(s) (system failure event), (2) graphically represents all levels of subevents and causes which are relative to creating the undesired event, and (3) qualitatively and/or quantitatively assesses the occurrence (unoccurrence) of the undesired event.

Design Element Listing

Line Number	Design Element	Design Element I.D. Number	Failure Mode	End Effect Number	Modal Failure Rate
1 . . .	O-Ring	0-1	Leakage	E001	.300
4 . . .	Spur Gear	G-1	Broken Tooth	E001	.002
8 . .	Ball Bearing	B-2	Misalignment	E001	.010

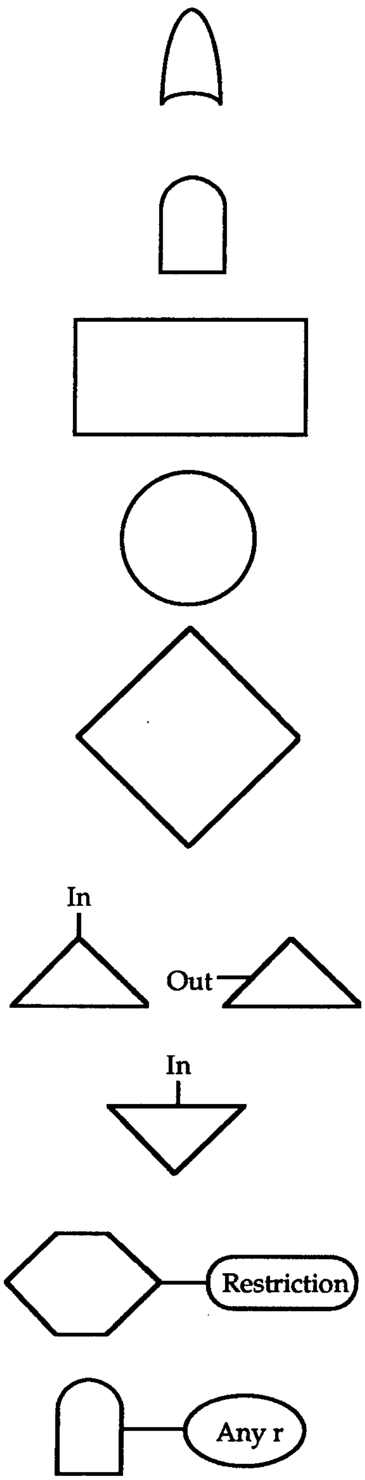
FIGURE 6.1-4: THE MAJOR HEADINGS IN THE DESIGN ELEMENT LISTING

End Effect Number	End Effect Description	Criticality Level	Total Modal Failure Rate	Design Element	Design Element I.D. Number	Failure Mode	Modal Failure Rate	Line Number
E001	Loss of Actuation	CSF	.312	O-Ring	0-1	Leakage	.300	1
				Ball Bearing	B-1	Misalignment	.010	8
				Spur Gear	G-1	Broken Tooth	.002	4

FIGURE 6.1-5: THE MAJOR HEADINGS IN THE EFFECTS LISTING

FTA provides an alternate failure mode evaluation technique to the FMECA discussed in Section 6.1. Recall that the FMECA starts at the lowest level of system configuration whereas the FTA begins at the top level. This is the primary difference in their approach to evaluating the failure modes of a system.

Fault tree construction begins with the identification of the undesired system failure event that forms the top event of the fault tree. The top event is then linked to its immediate causes and/or sub-events using the appropriate logic symbols shown in Figure 6.2-1. Linking of sub-events and causes continues in turn until the desired basic cause level is reached.



<u>OR Gate</u>
Shows a logic OR relationship. The output event will happen if and only if one or more of the input events happen.

<u>AND Gate</u>
Shows a logical AND relationship. The output event will happen if and only if all of the input events happen.

<u>Intermediate Event</u>
Describes a condition or event that is caused by the combination of fault causes through the input gate.

<u>Basic Event</u>
Shows the failure of an elementary component. No further development is needed.

<u>Undeveloped Event</u>
Shows an input event which could be developed further if more information were available. Could be used to show the failure of a subsystem which is considered to be a basic input event.

<u>Transfer Symbol</u>
Shows where one branch of the fault tree connects to another branch, and often used from one sheet to another. Connection to the apex means input to the tapped event; connection to the side means output from the tapped event. Upside down means the transferred branch is similar to the tapped event. The number in the triangle identifies the connections.

<u>INHIBIT Gate</u>
Shows that the output event will only occur if the input event happens in such a way that some restriction is satisfied.

<u>Combination (COM) Gate</u>
Shows that the output event will happen if any r out of the n input events happen.

FIGURE 6.2-1: SYMBOLS USED IN FTA

As an example, a fault tree was developed for a torque wrench design which is shown in Figure 6.2-2. The undesired event was specified as "the wrench applies the wrong torque." The fault tree resulting from this undesired event is shown in Figure 6.2-3.

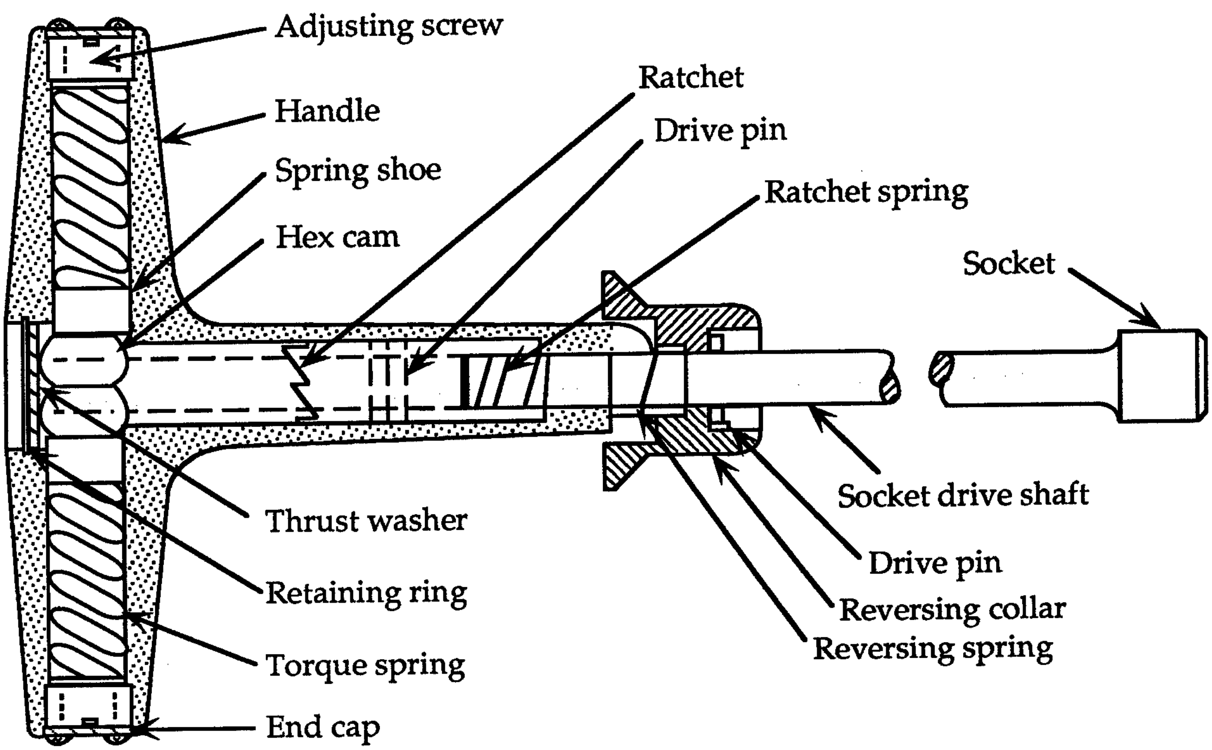


FIGURE 6.2-2: TORQUE WRENCH DESIGN [67]

Evaluation of the fault tree may be qualitative or quantitative. Qualitative analysis determines the combinations of basic failures that lead to the undesired event and the combinations of the complementary events to assure that the top event will not occur. A qualitative analysis is usually carried out when no elementary-level reliability data is available or where the primary objectives of the FTA can be fulfilled without quantifying the results.

In a quantitative analysis the probability of occurrence of the top event is calculated using the quantitative information of the basic events. Various quantitative analysis methods have been discussed in the literature (References [54], [59] and [65]) for evaluating the fault tree.

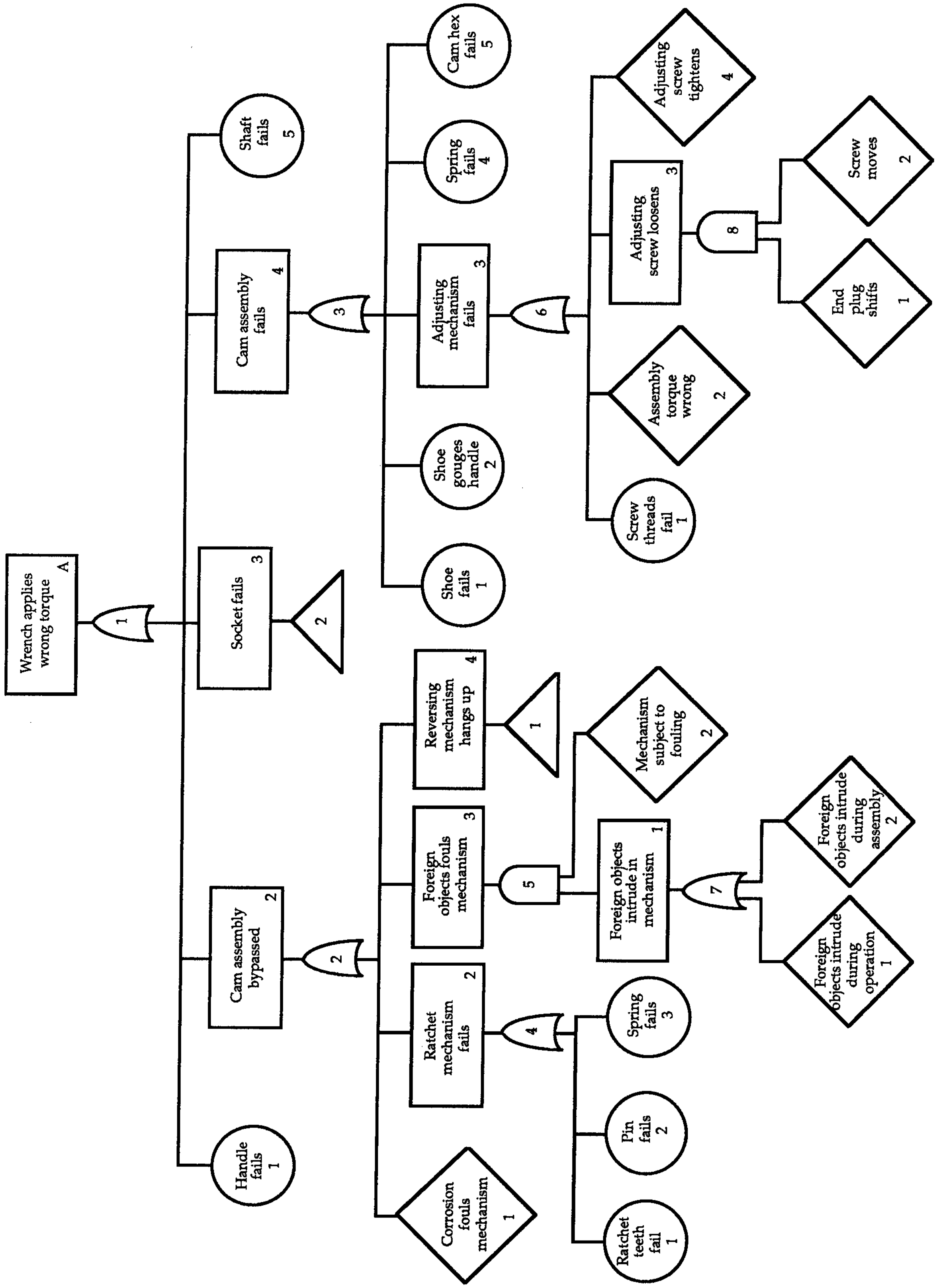


FIGURE 6.2-3: TORQUE WRENCH FAULT TREE [67]

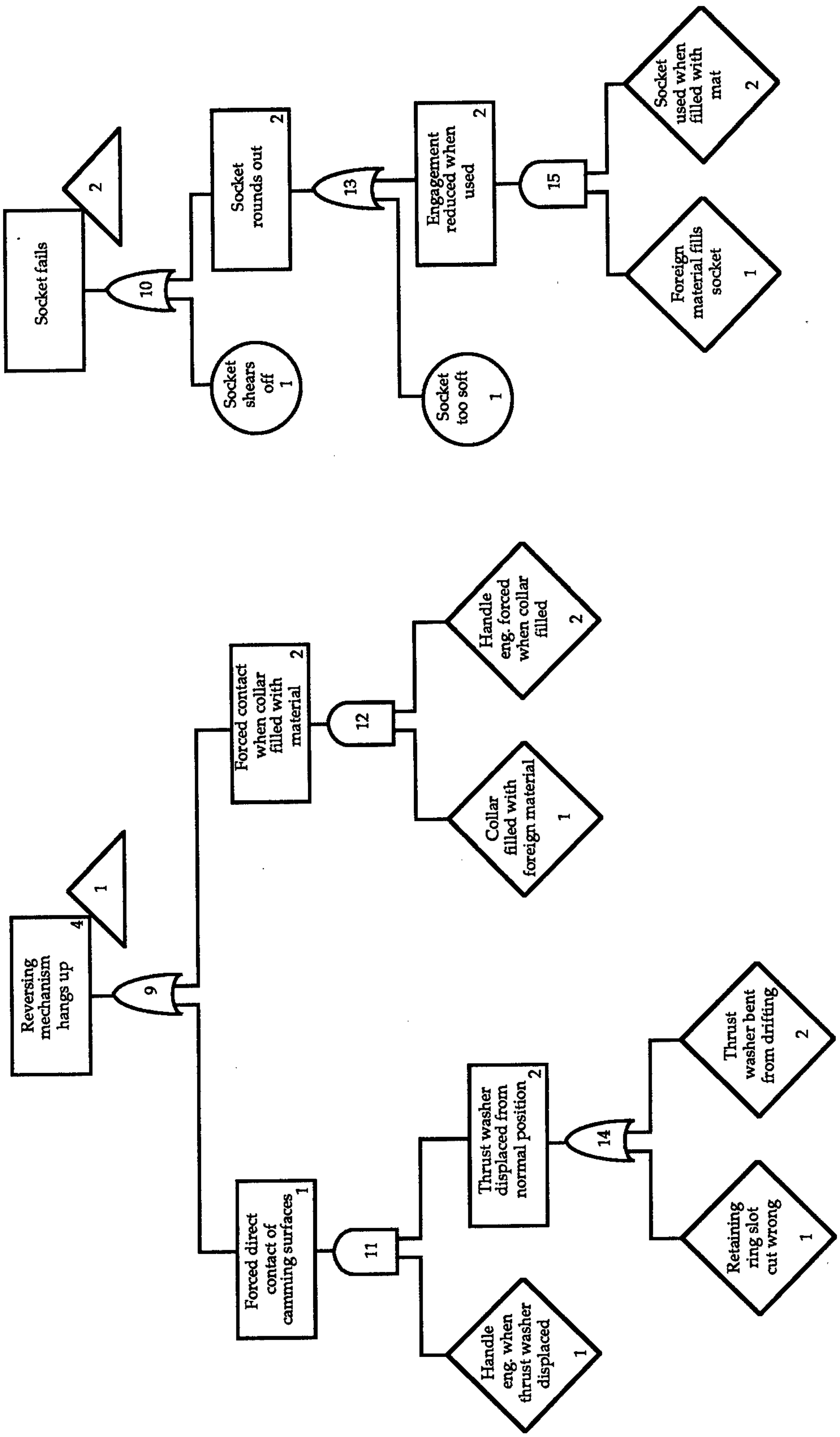


FIGURE 6.2-3: TORQUE WRENCH FAULT TREE [67] (CONT'D)

One possible method for quantitatively assessing the probability of occurrence of the top event is by propagating the probabilities of occurrence of the basic events upward through the fault tree. This method requires that the probability of occurrence of all basic events is known.

The combining of these probabilities follows the basic laws of probability given in Appendix B. The AND gate (refer to Figure 6.2-4 and Rule 3 of Appendix B) dictates that the following probability rule be applied to obtain the AND gate output probability (assuming all fault events are independent):

$$\Pr(A_1 \text{ and } A_2 \text{ and} \ldots A) = \prod_{i=1}^{n} \left[\Pr(A_i)\right] = \Pr(A_1)\Pr(A_2)\ldots\Pr(A_n) \qquad (6\text{-}3)$$

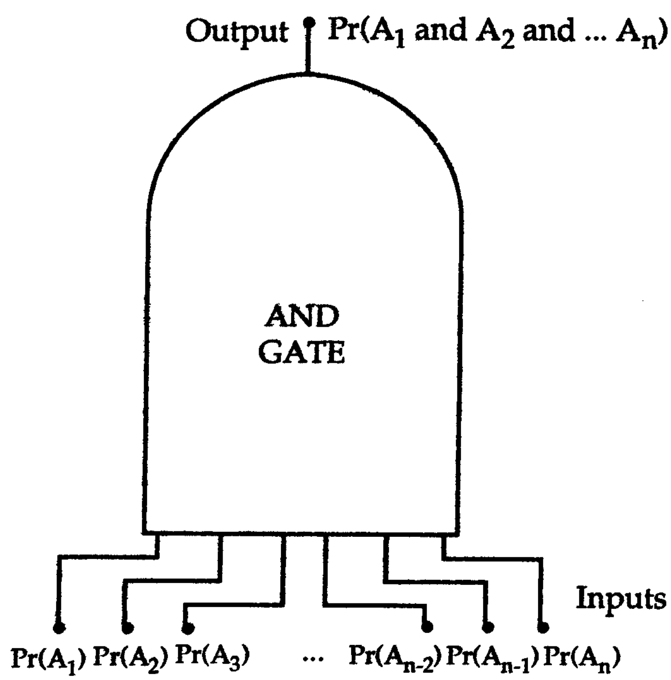


FIGURE 6.2-4: AND GATE INPUT, OUTPUT

The OR gate (refer to Figure 6.2-5 and Rule 5 or 7 of Appendix B) requires that one of the following probability rules be applied to obtain the OR gate output probability:

Inputs mutually exclusive:

$$\Pr(A_1 \text{ or } A_2 \text{ or} \ldots A_n) = \sum_{i=1}^{n} \Pr(A_i) \tag{6-4A}$$

Inputs <u>not</u> mutually exclusive:

$$\Pr(A_1 \text{ and / or } A_2 \text{ and / or} \ldots \text{ and / or } A_n) = 1 - \prod_{i=1}^{n} \left[1 - \Pr(A_i)\right] \tag{6-4B}$$

where,

$\Pr(A_i)$ = probability of occurrence of the ith input event (an input event may be represented as an event block, basic component fault or undeveloped event which are shown in Figure 6.2-1)

n = number of input events which may or may not be mutually exclusive

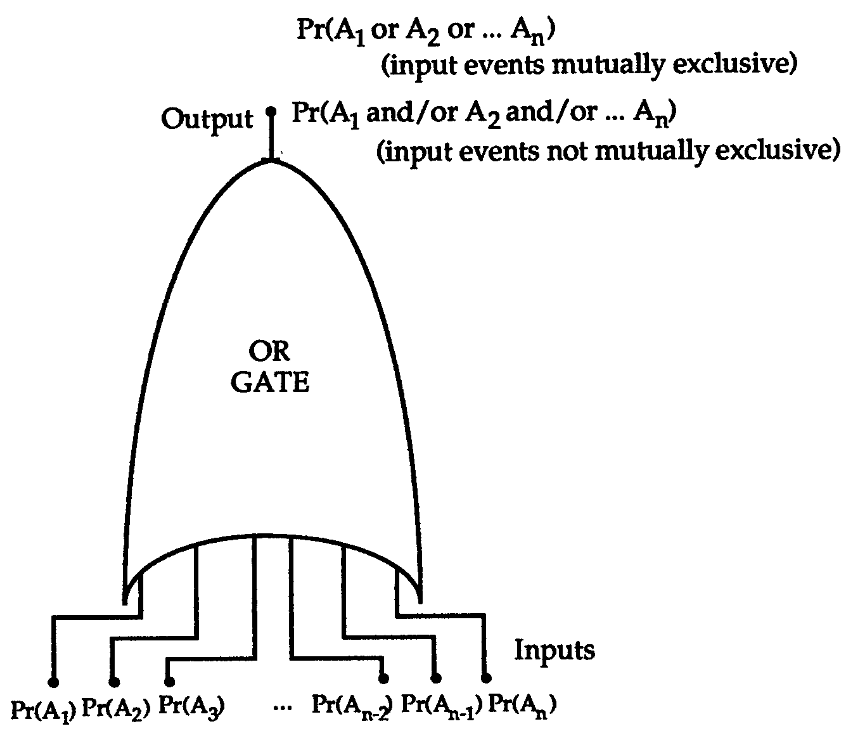


FIGURE 6.2-5: OR GATE INPUT, OUTPUT

Consider as an example the coolant supply system shown in Figure 6.2-6 from which the fault tree shown in Figure 6.2-7 is developed. The undesired event is identified as a loss of minimum flow to the heat exchanger. When the propagation of the probabilities of occurrence is applied to the fault tree of Figure 6.2-7, orderly calculations are performed to determine the probability of occurrence of the undesired event. The calculation procedure is initiated at the elementary level of the fault tree. The output of each gate is then determined from the gate type and the gate inputs. Equations (6-3) and (6-4) are applied in the following sequence to determine the probability of occurrence of the undesired event. The notation follows that identified in Figure 6.2-7.

1) $Pr(X_8) = 1 - [1 - Pr(X_{14})][1 - Pr(X_{15})][1 - Pr(X_{16})]$
2) $Pr(X_5) = 1 - [1 - Pr(X_8)][1 - Pr(X_9)]$
3) $Pr(X_7) = Pr(X_{12})\,Pr(X_{13})$
4) $Pr(X_4) = 1 - [1 - Pr(X_7)][1 - Pr(X_6)]$
5) $Pr(X_2) = 1 - [1 - Pr(X_3)][1 - Pr(X_4)][1 - Pr(X_5)]$
6) $Pr(A) = 1 - [1 - Pr(X_2)][1 - Pr(X_1)]$

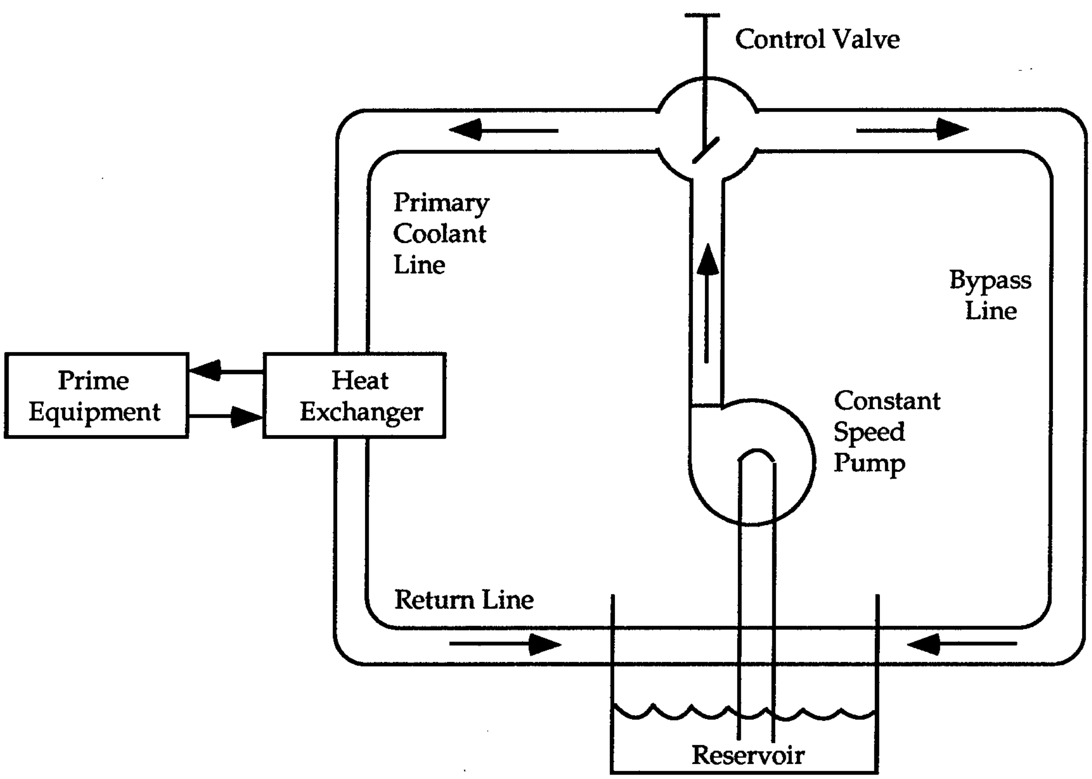


FIGURE 6.2-6: COOLANT SUPPLY SYSTEM [58]

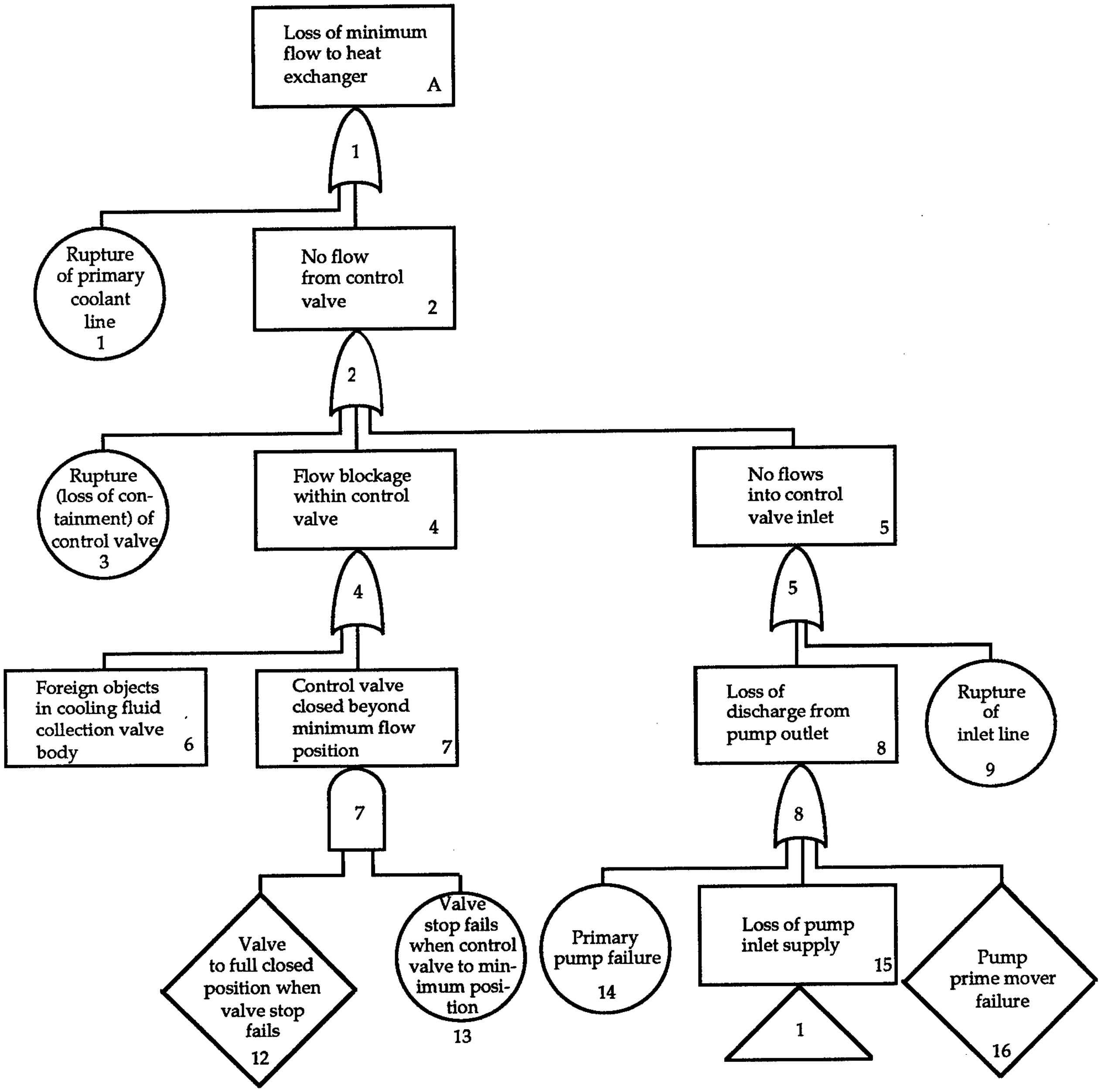


FIGURE 6.2-7: FAULT TREE FOR COOLANT SUPPLY SYSTEM [58]

7.0 CONCLUSION

It is important that engineering and management personnel develop an awareness of the quantitative mechanical reliability evaluation tools which are available today. Reliability evaluation techniques represent useful design tools and management aides for improving overall system performance. Mechanical Applications In Reliability Engineering represents a serious effort to collect and present only those practical mechanical part and system reliability evaluation techniques which have proven useful application. Many other valuable reliability techniques currently exist which have not been discussed here. We strongly encourage analysts to keep a constant vigal to discover and effectively utilize all of these new tools.

Reliability engineering has grown from a need by engineering and management personnel to quantitatively examine the "quality over the long run" of parts and systems. As the quantity of historical mechanical reliability data increases and is analyzed, the more effective mechanical reliability evaluation techniques can become in improving engineering designs. To make the current models more accurate, it is extremely important that mechanical reliability data (e.g., times-to-failure, failure mode probability of occurrence, probabilistic material characteristics and system failure process characteristics) be collected for statistical analysis. The Reliability Analysis Center maintains an extensive mechanical reliability data base/library where mechanical reliability data is collected, analyzed and summarized. The RAC encourages all organizations to utilize and/or contribute to this growing source of information.

Research and development of new engineering approaches to improve part/system performance is continually being developed in such areas as:

- finite element analysis
- cumulative damage analysis
- fatigue analysis
- optimization methods
- probabilistic design
- system reliability

In order for us to effectively apply the theories and tools generated from these state-of-the-art research areas to evaluate and improve the reliability of parts and systems, we must understand basic reliability practices and continue to be informed. This is accomplished by reading reliability periodicals, attending R&M conferences/symposiums, and ultimately evaluating the thoughts of the "many" until personal understanding is achieved. In this endeavor we wish you the best of luck!

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Appendix A:

Glossary of Terms for Reliability and Statistics

α (Alpha)	The characteristic life of the Weibull distribution. 63.2% of the lifetimes will be less than the characteristic life regardless of the value of β, the Weibull slope parameter.
Assessment	The use of test data and/or operational service data to form estimates of population parameters and to evaluate the precision of those estimates (synonym - Estimation).
β (Beta)	The parameter of the Weibull distribution that determines its shape and that implies the failure mode characteristic (infant mortality, random, or wearout). It is also called the slope parameter because it is estimated by the slope of the straight line on Weibull probability paper.
Confidence Coefficient	A measure of assurance that a statement based upon statistical (frequency) data is correct. The probability that an unknown parameter lies within a stated interval or is greater than or less than some stated value.
Confidence Interval	A region within which an unknown parameter is said to lie with stated probability. The region is two-sided when both upper and lower limits are specified. It is one-sided when only the upper or the lower limit is specified.
Confidence Limit	A bound of a confidence interval.
Constant Failure Rate	Characterizes a part with constant Hazard Rate, h(t). $h(t) = \text{constant} = \lambda$
Criticality	A relative measure of the consequences of a failure mode and its frequency of occurrence.
Criticality Analysis (CA)	A procedure by which each potential failure mode is ranked according to the combined influences of severity and probability of occurrence.
Cumulative Distribution Function, F(x)	The probability F(x) that a random variable X takes a value less than or equal to x.

Decreasing Force of Motality	Characterizes a part with decreasing hazard rate. This may occur, for instance, during the early portion of part life as indicated by the "bath tub" curve for parts.
Evaluation	A broad term used to encompass prediction, measurement, and demonstration.
Exponential Distribution	A probability distribution having the density function $f(x) = \lambda e^{-\lambda x}$ where λ is constant.
Failure	Performance below a specified minimum level or outside a specified tolerance interval.
Failure Effect	The consequence(s) a failure mode has on the operation, function or status of an item. Failure effects are classified as primary or local effects, secondary or next - level effects and system or end effects.
Failure Mechanism	A description of the failure process.
Failure Mode	The manner by which a failure is observed. Generally describes the way the failure effects the function of a part.
Failure Modes and Effects Analysis (FMEA)	A procedure by which each potential failure mode in a system is analyzed to determine the results or effects on the system and to classify each potential failure mode according to its severity.
Hazard Rate, h(t)	Also called the force of mortality (FOM) represents the probability that an item still functioning at time x will fail in the interval $(x, x + \Delta x)$, where Δx is an infinitesimal time increment. The hazard rate is <u>not</u> a density function. The hazard rate is defined by: $\frac{f(x)}{1 - F(x)}$
Increasing Force of Mortality	Characterizes a part with increasing hazard rate. This may occur, for instance, during the later portion of part life as indicated by the "bath tub" curve for parts.
Infant Mortality	A failure mode characterized by a hazard rate that decreases with age, i.e., new units are more likely to fail than old units.

Indenture Levels	The item levels which identify or describe relative complexity of assembly or function. The levels progress from more complex (major system) to the simpler (part) divisions.
Item	It is a nonspecific term used to denote any product, including systems, subsystems, sets, groups, assemblies, subassemblies, parts, materials, accessories, and so forth.
Log Normal Distribution	Statistical distribution which characterizes times to failure of terms displaying normally distributed logarithms of times to failure.
Mean, μ	The first moment of a probability distribution about its origin, the expected value of a random variable. The mean is the most commonly used measure of central tendency. Estimated by an arithmetic average.
Mean-Time-Between-Failure (MTBF)	A basic measure of reliability for repairable systems which follow the Homogeneous Poisson Process.
Mean-Time-To-Failure (MTTF)	The mean time to failure. If $f_P(x)$ is the Probability Density Function of random variable, P, then MTTF $= \int_0^\infty x\, f_P(x)\, dx$.
Monte Carlo Simulation	A mathematical model of a system with random elements, usually computer-adapted, whose outcome depends on the application of randomly generated numbers.
Normal Distribution	The most prominent continuous distribution in statistics, frequently referred to as the Gaussian or bell-shaped distribution. Its density function is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \; -\infty < x < \infty$ with mean, μ, and variance, σ^2. The theoretical justification for the normal distribution lies in the central-limit theorem, which shows that under very broad conditions the distribution of the average of n independent observations from any distribution approaches a normal distribution as n becomes large.

Normal Variable	A random variable that is normally distributed.
Part Failure Rate, λ_p	The constant hazard rate of parts whose failure model is exponential.
Probability Density Function, f(x)	It is a continuous function of a random variable, X, such that its integral $\int_a^b f(x)\,dx$ represents the probability of x assuming a value between a and b. The integral over all x is equal to 1.
Random	An event that is independent of time, in the sense that an old unit is as likely to fail as a new unit. In other words, the hazard rate remains constant with age.
Random Variable	An output of an experiment which may take any of the values of a specified set with a specified relative frequency or probability.
Redundancy	The existence of more than one means for accomplishing a given function. All means of accomplishing the function need not necessarily be identical.
Redundancy, Active	The redundancy wherein all redundant items are operating simultaneously.
Redundancy, Standby	That redundancy wherein the alternative means of performing the function is inoperative until needed and is switched on upon failure of the primary means of performing the function.
Reliability	The probability of failure free operation over a time interval under stated conditions given that it is operable at the beginning of the interval.
Reliability Engineering	A professional discipline which combines knowledge of statistics and engineering for the purpose of quantitatively evaluating, prediction, measuring and improving the reliability of human products.

Variance, σ^2

The second moment about the mean of a probability distribution. A measure of the dispersion of random variable about its mean value. In testing variance is a measure of random errors in a series of measurements.

Wearout (of parts)

A part failure mode characterized by a hazard rate that increases with age, i.e., old parts are more likely to fail than new parts.

Weibull Distribution

The statistical distribution modeled by the probability density function:

$$f(x) = \frac{\beta}{\alpha^\beta} x^{\beta-1} \exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right] x > 0, \alpha > 0, \beta > 0$$

The function was introduced in 1951 by W. Weibull on empirical grounds based on studies of material strength.

Appendix B:

Probability Laws

Rule 1:

If $\Pr(A)$ and $\Pr(\overline{A})$ represent respectively the probability of the event A occurring and not occurring, then:

$$\Pr(\overline{A}) = 1 - \Pr(A) \tag{B.1}$$

Rule 2:

If A and B are two independent events, then the probability that both A and B will happen, known as their joint probability, is the product of their respective individual probabilities - that is:

$$\Pr(A \text{ and } B) = \Pr(AB) = \Pr(A)\Pr(B) \tag{B.2}$$

Rule 3:

The probability of the joint occurrence of each of N independent events $A_1, A_2, \ldots, A_N$ is the product of their individual probabilities - that is:

$$\Pr(A_1 \text{ and } A_2 \text{ and } \ldots A_N) = \Pr\left(\prod_{i=1}^{N} A_i\right) = \Pr(A_1)\Pr(A_2)\ldots\Pr(A_N) \tag{B.3}$$

Rule 4:

If A and B are two mutually exclusive events - that is, $\Pr(AB) = 0$ - then the probability that one of these two events will take place is given by the sum of their individual probabilities:

$$\Pr(A \text{ or } B) = \Pr(A+B) = \Pr(A) + \Pr(B) \tag{B.4}$$

Rule 5:

The probability of occurrence of one of N mutually exclusive events $A_1, A_2, ..., A_N$ is:

$$\Pr(A_1 \text{ or } A_2 \text{ or } ... A_N) = \Pr\left(\sum_{i=1}^{N} A_i\right) = \sum_{i=1}^{N} \Pr(A_i) \tag{B.5}$$

Rule 6:

If A and B are two events that are not necessarily mutually exclusive - that is, if $\Pr(AB) \neq 0$ - then the probability that at least one of these two events will take place is given by the sum of their individual probabilities less their joint probability:

$$\Pr(A \text{ and / or } B) = \Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{B.6}$$

Rule 7:

The probability that at least one of N events $A_1, A_2, ..., A_N$ will take place is:

$$\begin{aligned} \Pr(A_1 \text{ and / or } A_2 \text{ and / or } ... A_N) &= \Pr\left(\sum_{i=1}^{N} A_i\right) \\ &= 1 - \Pr\left(\overline{A}_1 \overline{A}_2 ... \overline{A}_N\right) \\ &= 1 - \prod_{i=1}^{N} \left[1 - \Pr(A_i)\right] \end{aligned} \tag{B.7}$$

Appendix C:

The Greek Alphabet

Greek Name	Greek Letter	
	Lower Case	Capital
Alpha	α	Α
Beta	β	Β
Gamma	γ	Γ
Delta	δ	Δ
Epsilon	ε	Ε
Zeta	ζ	Ζ
Eta	η	Η
Theta	θ	Θ
Iota	ι	Ι
Kappa	κ	Κ
Lambda	λ	Λ
Mu	μ	Μ
Nu	ν	Ν
Xi	ξ	Ξ
Omicron	ο	Ο
Pi	π	Π
Rho	ρ	Ρ
Sigma	σ	Σ
Tau	τ	Τ
Upsilon	υ	Υ
Phi	φ	Φ
Chi	χ	Χ
Psi	ψ	Ψ
Omega	ω	Ω

Appendix D:

Descriptive Statistics

This appendix presents important descriptive statistics for both the population and sample. The population is defined as a collection of objects having common characteristics. The population is typically viewed as the universe or collection of all such objects. The parameters of the population are fixed, though usually unknown. A sample is a subset of the population. The characteristics of a sample can vary from sample to sample even though extracted from the same population. These are two conceptually significant items when discussing descriptive statistics.

A. Measures of Central Tendency

Mean:

Population Mean (continuous random variable): $\mu = E(x) = \int_{-\infty}^{\infty} x\, f(x)dx$

Sample Mean: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Median:

Population Median: $0.5 = \int_{-\infty}^{x_{50}} f(x)dx$ or $F(x) = 0.5$

Sample Median:

1) The middle number from order statistics when the sample size is odd

2) The average of the two middle numbers from order statistics when the sample size is even

Mode:

Population Mode: $\dfrac{df(x)}{dx} = 0$

Sample Mode: Value of Random Variable = Max (Prob. of occur.)

B. Measures of Variability

Variance:

Population Variance (continuous random variable): $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$

Sample Variance: $S^2 = \dfrac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n-1}$ (unbiased form)

Standard Deviation:

Population Standard Deviation (continuous random variable): $\sigma = \sqrt{\sigma^2}$

Sample Standard Deviation: $S = \sqrt{S^2}$

Appendix E:

Gamma Function

Gamma Function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \quad \text{for } 1 \leq x \leq 2$$

[For other values use the formula $\Gamma(x + 1) = x\Gamma(x)$]

x	Γ(x)
1.00	1.00000
1.01	.99433
1.02	.98884
1.03	.98355
1.04	.97844
1.05	.97350
1.06	.96874
1.07	.96415
1.08	.95973
1.09	.95546
1.10	.95135
1.11	.94740
1.12	.94359
1.13	.93993
1.14	.93642
1.15	.93304
1.16	.92980
1.17	.92670
1.18	.92373
1.19	.92089
1.20	.91817
1.21	.91558
1.22	.91311
1.23	.91075
1.24	.90852
1.25	.90640
1.26	.90440
1.27	.90250
1.28	.90072
1.29	.89904
1.30	.89747
1.31	.89600
1.32	.89464
1.33	.89338
1.34	.89222
1.35	.89115
1.36	.89018
1.37	.88931
1.38	.88854
1.39	.88785
1.40	.88726
1.41	.88676
1.42	.88636
1.43	.88604
1.44	.88581
1.45	.88566
1.46	.88560
1.47	.88563
1.48	.88575
1.49	.88595

x	Γ(x)
1.50	.88623
1.51	.88659
1.52	.88704
1.53	.88757
1.54	.88818
1.55	.88887
1.56	.88964
1.57	.89049
1.58	.89142
1.59	.89243
1.60	.89352
1.61	.89468
1.62	.89592
1.63	.89724
1.64	.89864
1.65	.90012
1.66	.90167
1.67	.90330
1.68	.90500
1.69	.90678
1.70	.90864
1.71	.91057
1.72	.91258
1.73	.91467
1.74	.91683
1.75	.91906
1.76	.92137
1.77	.92376
1.78	.92623
1.79	.92877
1.80	.93138
1.81	.93408
1.82	.93685
1.83	.93969
1.84	.94261
1.85	.94561
1.86	.94869
1.87	.95184
1.88	.95507
1.89	.95838
1.90	.96177
1.91	.96523
1.92	.96877
1.93	.97240
1.94	.97610
1.95	.97988
1.96	.98374
1.97	.98768
1.98	.99171
1.99	.99581
2.00	1.00000

Appendix F:

RAC Products

RAC Product Order Form

Ordering Code	Title	U.S. Price	Non-US Price	Qty.	Item Total
Concurrent Engineering Series					
ITCE	Introduction to Concurrent Engineering	$75	$85		
WCCA	Worst Case Circuit Analysis Application Guidelines	$75	$85		
FMECA	Failure Mode, Effects and Criticality Analysis	$75	$85		
FTA	Fault Tree Analysis Application Guide	$75	$85		
Data Publications					
NPRD	Nonelectronic Parts Reliability Data	$150	$170		
FMD	Failure Mode/Mechanism Distributions	$100	$120		
ECDS	Environmental Characterization Device Sourcebook	$100	$120		
NONOP-1	Nonoperating Reliability Databook *Price Reduced*	$50	$60		
VZAP	Electrostatic Discharge Susceptibility Data	$150	$170		
MDR-22	Microcircuit Screening Analysis *Price Reduced*	$50	$60		
Application Guides					
RMST	Reliability & Maintainability Software Tools	$50	$60		
SLEA	Service Life Extension Assessment	$50	$60		
SOAR-6	ESD Control in the Manufacturing Environment *Price Reduced*	$50	$60		
SOAR-4	Confidence Bounds for System Reliability	$50	$60		
TEST	Testability Design and Assessment Tools	$50	$60		
PRIM	A Primer for DoD Reliability, Maintainability, Safety & Logistic Standards *Price Reduced*	$100	$120		
NPS	Mechanical Applications in Reliability Engineering	$100	$120		
QREF	RAC Quick Reference Guides *Price Reduced*	$25	$35		
RMIMP	R&M Implications of Current DoD Acquisition Policy/Procedures	$50	$60		
TOOLKIT	RL Reliability Engineer's Toolkit - 2nd Edition	$12	$22		
RDSC	The Reliability Sourcebook - "How and Where to Obtain R&M Data and Information"	$50	$60		
SOAR-2	Practical Statistical Analysis for the Reliability Engineer	$50	$60		
Component Publications					
MFAT-1	Microelectronics Failure Analysis Techniques: A Procedural Guide *Price Reduced*	$70	$80		
ATH	Analog Testing Handbook	$100	$120		
PEM	Plastic Microcircuit Packages: A Technology Review	$50	$60		
GAAS	An Assessment of Gallium Arsenide Device Quality and Reliability	$50	$60		
MFAT-2	GaAs Microcircuit Characterization and Failure Analysis Techniques: A Procedural Guide *Price Reduced*	$50	$60		
MFAT 1 & 2	Combined set of MFAT-1 and MFAT-2 *Price Reduced*	$100	$120		
QML	Qualified Manufacturer's List: New Device Manufacturing and Procurement Technique	$50	$60		
Reliable Application of Components					
PSAC	Parts Selection, Application and Control	$75	$85		
CAP	Reliable Application of Capacitors	$50	$60		
HYB	Reliable Application of Hybrids	$50	$60		
Total Quality Management					
SOAR-7	A Guide for Implementing Total Quality Management	$75	$85		
TQM	TQM Toolkit	$75	$85		
SOAR-8	Process Action Team Handbook *Price Reduced*	$40	$50		
Computer Products					
NPRD-P	NPRD-91 PC Version	$400	$440		
VZAP-P	VZAP-91 PC Version	$400	$440		
NRPS	Nonoperating Reliability Prediction System *Price Reduced*	$700	$740		
VPRED	VHSIC Reliability Prediction Software *Price Reduced*	$100	$120		
217N1	MIL-HDBK-217F, Notice 1	$75	$85		
217N2D	MIL-HDBK-217F, Notice 2 (Draft)	$75	$85		
338D	MIL-HDBK-338B (Draft)	$95	$105		
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				Quantity Discount - See Right	
				Order Total	

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